Proceedings of the East Asian School
of Knots, Links, and Related Topics
February 16–20, 2004 • Seoul, Korea

ON THE JONES POLYNOMIALS OF RIBBON KNOTS OF 1-FUSION

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In [5], T.Sakai gives a relation between the first derivative at -1 and the third derivative at 1 of the Jones polynomial of 6_1-like ribbon knot, which is a special class of ribbon knots of 1-fusion.

Theorem ([5]). Let K be a 6_1-like ribbon knot. Then the following holds.

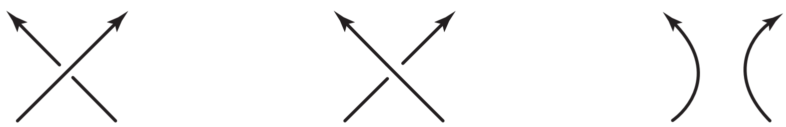
$$2J'''_K(1) = -9J'_K(-1) - 72.$$

In this talk we give a formula for the first derivative at -1 of the Jones polynomial of ribbon knots of 1-fusion, which is an extension of a formula obtained by Sakai for 6_1-like ribbon knots in [5].

Definition. The Jones polynomial $J_L(t) \in \mathbb{Z}[t^{1/2}, t^{-1/2}]$ is an invariant of an oriented link L in S^3, which is defined by the following formulas:

$$t^{-1}J_{L_+}(t) - tJ_{L_-}(t) = (t^{1/2} - t^{-1/2})J_{L_0}(t),$$
$$J_O(t) = 1;$$

where L_+, L_-, L_0 are three oriented links, which are identical except near one point where they are as shown in the figure and O denotes the trivial knot ([1]).



L+ L− L0

Definition. A *ribbon disk* is an immersed 2-disk of D^2 into S^3 with only transverse double points such that the singular set consists of ribbon singularities, that is, the preimage of each ribbon singularity consists of a properly embedded arc in D^2 and an embedded arc interior to D^2. A knot is a *ribbon knot* if it bounds a ribbon disk in S^3 (cf. [2]).

Definition. We call a knot K in S^3 a ribbon knot of *1-fusion*, if it has a knot diagram $\tilde{K}$ as described in Figure 1, where n is even and each small rectangle named C_i is determined by $c_i \in \{-1, 0, +1\}$ $(i = 1, 2, \cdots, n)$ and there are disjointly embedded $(n+1)$ bands in the big rectangle, being knotted, twisted and mutually linked (cf. [4]). We call the diagram $\tilde{K}$ *1-fusion diagram* of K. $\tilde{K}$ gives a ribbon disk bounded by K. The Alexander polynomial of K is determined by $c_1, c_2, \cdots, c_n$ ([3]).

Let α_i $(i = 1, 2, \cdots, n+1)$ denote the twisting number of i-th band in the big rectangle, and let $\alpha_{i,j}$ $(i < j)$ denote the relative linking number of i-th band and j-th band in the big rectangle. That is: Direct the bands from left to right and attach a sign to each crossing of different bands, as shown in Figure 2. Then $\alpha_{i,j}$ is half the sum of the signs of the crossings of i-th and j-th band.

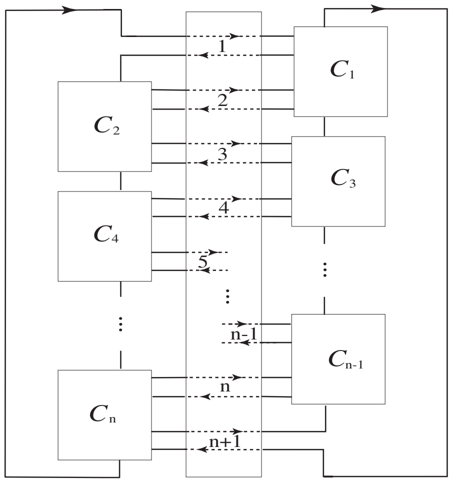


Figure 1: $\tilde{K}$

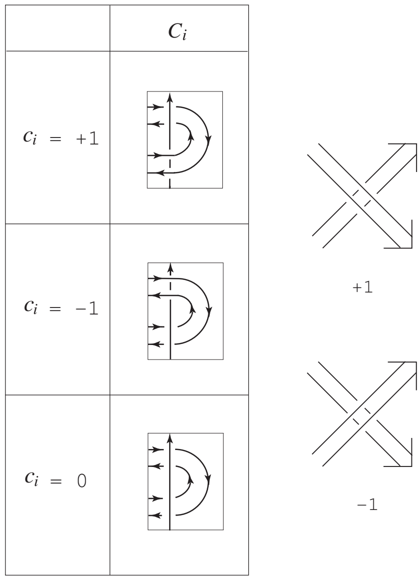


Figure 2

Theorem 1. *Let K and $\tilde{K}$ be as above, and let $J_K(t)$ be the Jones polynomial of K. Then $J'_K(-1)$ is a linear expression of α_i and $\alpha_{i,j}$:*

$$J'_K(-1) = \sum_{1\leq i\leq n+1} A_i\alpha_i + \sum_{1\leq i<j\leq n+1} A_{i,j}\alpha_{i,j} + B,$$

where each A_i, $A_{i,j}$ and B is expressed by $c_1, c_2, \cdots, c_n$.

Remark 1. A knot K which has the 1-fusion diagram with $(c_1, c_2) = (+1, +1)$ as shown in Figure 3.0 is called 6_1-like ribbon knot by Sakai in [5]. Then Sakai shows

$$J'_K(-1) = 16(\alpha_1 + \alpha_2 + \alpha_3 - \alpha_{1,2} - \alpha_{1,3} - \alpha_{2,3}) - 8.$$

Note that the Alexander polynomial of K is $-2t^{-1} + 5 - 2t$.

Example 1. We call a knot K KT-like ribbon knot, if it has the 1-fusion diagram with $(c_1, c_2, c_3, c_4) = (+1, +1, 0, -1)$ as shown in Figure 3.1. Then by Theorem 2, we have

$$J'_K(-1) = 48(\alpha_{2,3} - \alpha_{2,4} - \alpha_{3,5} + \alpha_{4,5}).$$

Note that the Alexander polynomial of K is 1.

Example 2. If K has the 1-fusion diagram with $(c_1, c_2, c_3, c_4) = (0, +1, +1, 0)$ as shown in Figure 3.2, then by Theorem 2, $J'_K(-1) = 0$. Note that the Alexander polynomial of K is 1.

So if K satisfies $J'_K(-1) \neq 0$, then K can not bound a ribbon disk shown in Figure 3.2.

Remark 2. For any Laurent polynomial $f(t)$ with $f(1) = \pm 1$, there exists a ribbon knot of 1-fusion whose Alexander polynomial is $f(t)f(t^{-1})$ ([3]).

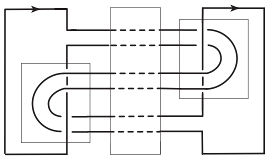


Figure 3.0

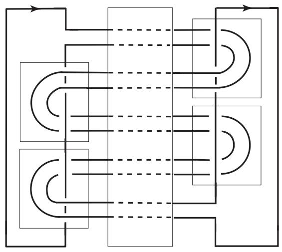


Figure 3.1

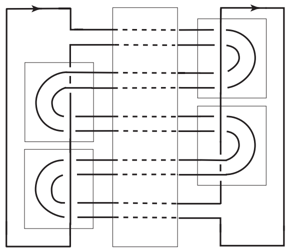


Figure 3.2

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