

STRONGLY n -TRIVIAL 2-BRIDGE KNOTS AND LINKS

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A knot K in S^3 is called *strongly n -trivial* if there exist $n + 1$ crossings contained in a diagram of K such that the result of any $0 < m \leq n + 1$ crossing changes on these crossings is the trivial knot. In [3], Howards–Luecke showed that if K is non-trivial and strongly n -trivial, then n is less than $6g(K) - 3$, where $g(K)$ is the genus of K . In particular, the trivial knot is the only knot that is strongly n -trivial for all n ([3, Corollary 1.5]). It is also known that Vassiliev invariants of orders $\leq n$ vanish for strongly n -trivial knots. Further, in [1], Askitas–Kalfagianni have had more algebraic estimations and restrictions of strong triviality of knots. (In [1], a strongly n -trivial knot is called $(n + 1)$ -*adjacent* to the trivial knot.) For example, they showed that for $n \geq 2$, the Alexander polynomial of strongly n -trivial knots is trivial ([1, Theorem 1.2]). For examlpe, we easily find that the trefoil knot and the figure-eight knot are strongly 1-trivial (Figure 1) and by the Askitas–Kalfagianni’s theorem, these knots fail to be strongly n -trivial for $n \geq 2$.

In this talk, we report on a theorem on strong triviality of 2-bridge knots which is proved by combining Dehn surgery techniques in [2], [6], [7], [8].

Our theorem is the following ([9]).

Theorem 1. *A 2-bridge knot is strongly n -trivial for $n \geq 1$ if and only if it is the trivial knot or the trefoil knot or the figure-eight knot.*

Remark 2. (i) The unknotting number of strongly n -trivial knots is one by definition. Actually, the proof of Theorem 1 is similar to Kanenobu–Murakami’s theorem in [5] which determines 2-bridge knots with unknotting number one.

(ii) The speaker does not know whether Theorem 1 can be obtained by purely algebraic methods.

(iii) A knot K is n -*adjacent* to a knot K' ($K \xrightarrow{n} K'$) if there exist n (generalized) crossings contained in a diagram of K such that the result of any $0 < m \leq n + 1$ crossing changes on these crossings is K' . Note that $K \xrightarrow{n} K'$ does not imply $K' \xrightarrow{n} K$. Kalfagianni–Lin have obtained deep results on n -adjacency of knots mainly by geometric arguments (ex. see [54]).

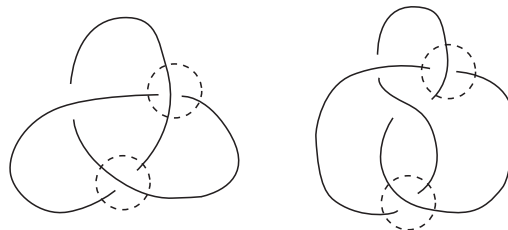


FIGURE 1

(iv) We may also consider strongly n -trivial *links*. For 2-bridge links, a theorem similar to Theorem 1 can be proved by a same argument [10]. Recently, however, Tsutsumi [11] announced that there are very strong geometric restrictions to strongly n -trivial links, which extensively include the 2-bridge link case. See his draft in this volume!

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