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COVERING SPACE OF S^3 BRANCHED OVER PRETZEL KNOT

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In 1982, W. Thurston [1] introduced the notion of the *universal*. A link or knot k in S^3 is universal, if every closed orientable 3-manifold can be represented as a covering space od S^3 branched over k. And he gave an example of universal link. After that, Hilden, Lozano, and Montesinos gave an example of universal knot, but it was very complicated. In 1987, they gave a necessary and sufficient condition for a two-bridge knot and link to be universal [2]. And in 1990, the auther gave a necessary and sufficient condition for a pretzel link of type $(a_1, a_2, \cdots, a_m)$ where at least two $a_i' s$ are even [3].

But generally speaking it is difficult to show that for a given knot k, it is universal or not, therefor, we will consider the following problem:

Which knot k in S^3 does have a S^3 as a covering space of S^3 branched over k?

If k is universal, then k has a 3-sphere as a branched covering space. And it is known that the following knots have, two-bridge knot, torus knot pretzel link.

Then we will consider for a pretzel knot has a 3-sphere or not.

Definition A pretzel knot is a knot consisting of 2-strand braid with $a_1-, a_2-, \cdots, a_m-$,half twists, which we denote by $p(a_1, a_2, \cdots, a_m)$. We assume that $a_i \neq 0$ for $i = 1, 2, \cdots, m$. $p(2, 3, -5)$ is shown in the following Figure.

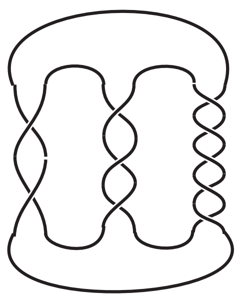

FIGURE 1. $p(2, 3, -5)$

We only consider $p = p(a_1, a_2, \cdots, a_m)$ is knot (that is p has only one component), only the following two cases ocuur:

(I) only one a_i is even,
(II) all of a_i's and m are odd.

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We say that p is even type (or odd type *resp.*), if it is of the case (I) (or case (II) *resp.*).

Theorem[with Ebrique Ramírez Losada (CIMAT)]
Every pretzel knot p of even type has a 3-sphere as a branched covering space branched over p.

Reference

[1] W. Thurston : *Universal links.* (preprint)
[2] H. Hilden, M. Lozano and J. Montesinos : *On knots that are univeral*, Topology, **24**(1985), 499-504.
[3] Y. Uchida, *Universal pretzel links*, Knots 90, Walter de Gruyter, Berlin, New York, (1992), pp.241-270