On diffeomorphisms over non-orientable surfaces standardly embedded in the 4-sphere

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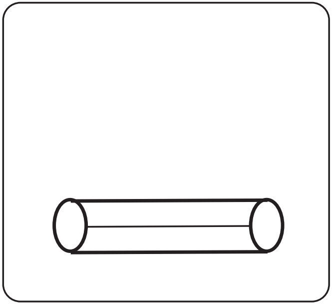
January 11, 2012

Introduction

Deform this annulus in S^4 with fixing boundary...



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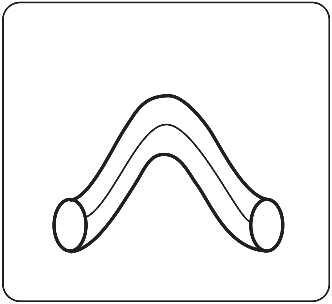

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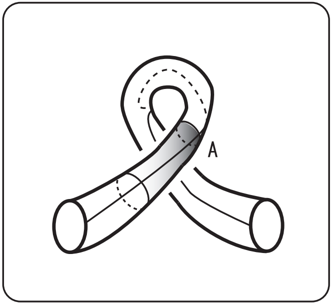

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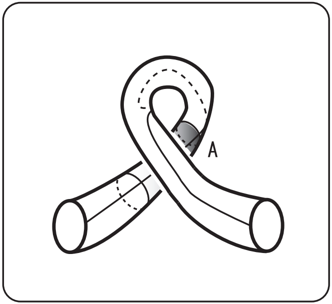

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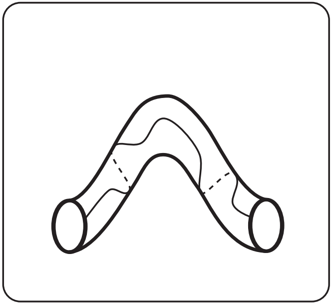

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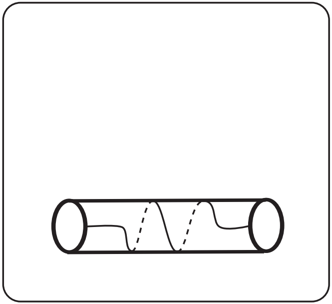

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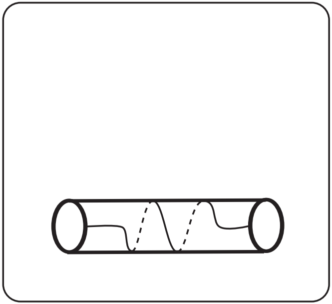

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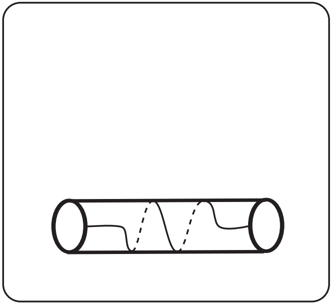

$\Rightarrow$ A double twist extends to the ambient S^4.

How many diffeomorphisms over surfaces embedded in a 4-dimensional space are extendable to the ambient space?

General setting

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such that

$$\begin{array}{ccc} S & \xrightarrow{e} & M \\ \phi \downarrow & & \downarrow \Phi \\ S & \xrightarrow{e} & M \end{array}$$

Known results

Σ_g = the orientable closed surface of genus g.
$i : \Sigma_g \hookrightarrow \mathbb{R}^3 \times \{0\} \subset \mathbb{R}^4 \subset \mathbb{R}^4 \cup \{\infty\} = S^4$: the standard embedding of Σ_g into S^4.

Theorem (Montesinos ($g = 1$) ; H ($g \geq 2$))

A diffeomorphism ϕ over the Σ_g is $i-$extentable to S^4
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$S \overset{e}{\hookrightarrow} M$ is flexible $\underset{\text{Def.}}{\Longleftrightarrow}$ every diffeomorphism over S is e-extendable.
Remark: Any embedding of a closed surface into S^4 is NOT flexible.

Theorem [A. Yasuhara, H, (2008)]

M: a smooth 4-manifold satisfying the following condition (*).

Condition(*) : For some 4-ball D in M, a Hopf link on ∂D^4 is the boundary of mutually disjoint two 2-disks embedded in $M - \mathrm{int} D^4$.

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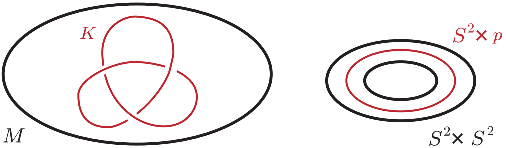
Since $\mathbb{C}P^2$, $S^2 \times S^2$ and $K3$-surface satisfy the condititon (*);

Corollary

For any closed surafce S, there is a flexible embedding of S into $\mathbb{C}P^2$, $S^2 \times S^2$ and $K3$-surface.

M : a simply connected smooth closed 4-manifold
S : a closed surface,
$S \overset{e}{\hookrightarrow} M$: a smooth embedding.
$(M\#S^2 \times S^2, e(S)\#S^2 \times \{p\})$: stabilization of e.

In the following figure, $K = e(S)$.



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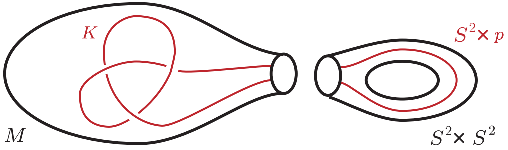

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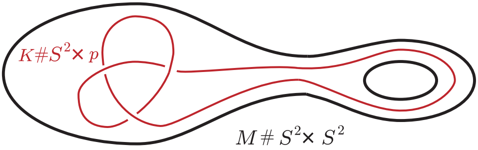

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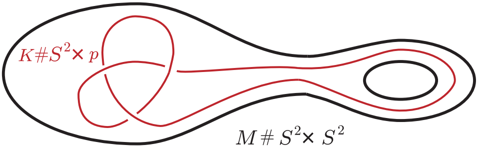


Theorem [A. Yasuhara, H, (2008)]

Under the above contidion, the stabilization of any embedding e is flexible.

Main topic of this talk

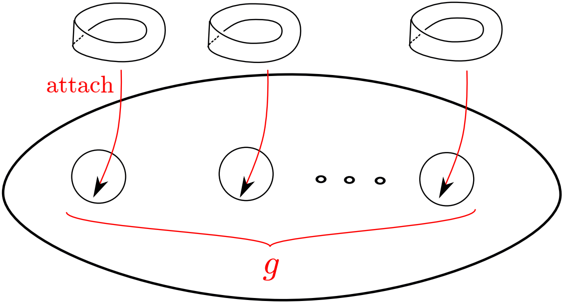
Consider the non-orientable analogy of the following Theorem.

Theorem [Montesinos ($g = 1$) ; H ($g \geq 2$)]

Let Σ_g be standardly embedded in S^4.
A diffeomorphism ϕ over the Σ_g is extentable to S^4
$\Leftrightarrow$ ϕ preserves the Rokhlin quadratic form of the Σ_g.

Setting

N_g: the non-orientable surface of genus g.



$N_g \overset{e}{\hookrightarrow} S^4$: a smooth embedding.

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C: an immersed circle on N_g,
D: a connected orientable surface immersed in S^4
s.t. (1) $\partial D = C$, (2) D is not tangent to N_g,
ν_D: the normal bundle of D.
$\nu_D|_C$ = a solid torus with the unique trivialization induced from any trivialization of ν_D
$N_{N_g}(C)$: the tubular neighborhood of C in N_g
$n(D)$ = the number of right hand half-twist of $N_{N_g}(C)$ with respect to the trivialization of $\nu_D|_C$.

$$q_e([C]) := n(D) + 2D \cdot F + 2Self(C) \mod 4,$$

where $D \cdot F = \mod 2$ intersection number of D and F,
$Self(C) = \mod 2$ double points number of C,

and 2 means an injection $\mathbb{Z}_2 \to \mathbb{Z}_4$ defined by $2[n]_2 = [2n]_4$.

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$q_e(x+y) = q_e(x) + q_e(y) + 2 < x, y >_2$, where $< x, y >_2$ means $\mod 2$ intersection number between x and y.

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cf: Y. Matsumoto, *An elementary proof of Rochlin's signature theorem and its extension by Guillou and Marin*, in "À la Recherche de la Topologie Perdue", Progress in Math., 62(1986), 119–139.

A diffeomorphism ϕ over N_g is $e-$extendable
$\underset{\text{Def.}}{\Longleftrightarrow}$ $\exists\Phi$: an orientation preserving self-diffeomorphism over S^4 such that

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Easy to see: ϕ is $e-$extendable $\Rightarrow$ ϕ preserves q_e.
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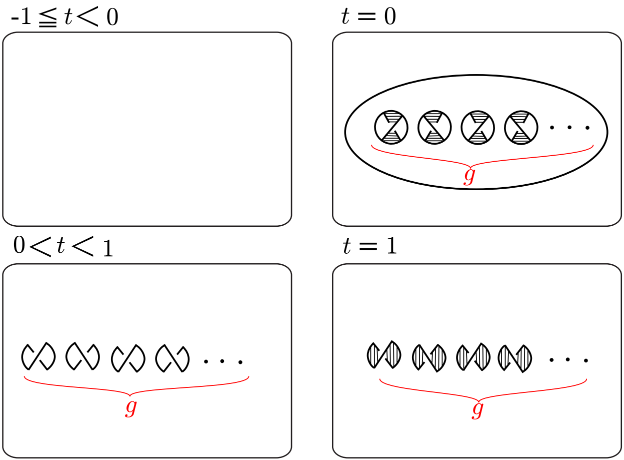
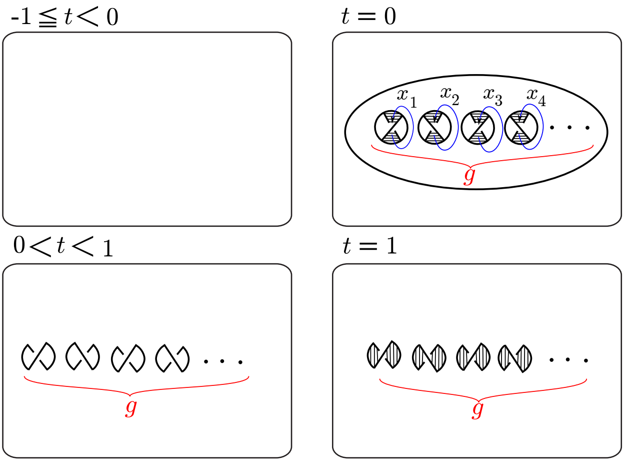
The answer depends on e.



The embedding $N_g \overset{os}{\hookrightarrow} S^4$ is o-standard $\underset{Def}{\Longleftrightarrow}$

Under the decomposition $S^4 = D^4_- \cup S^3 \times [-1,1] \cup D^4_+$, the image of os is in $S^3 \times [-1,1]$ and as shown in the above figure.



For the basis $\{x_1, \cdots, x_g\}$ for $H_1(N_g, \mathbb{Z}_2)$ shown in the above figure, $q_{os}(x_{2i-1}) = +1, q_{os}(x_{2i}) = -1, < x_i, x_j >_2 = \delta_{i,j}$.

Main Theorem [H. (arXiv:1109.1668)]

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$\Leftarrow$???
The diffeomorpshims ϕ_1 over N_g is extendable, and ϕ_1 and ϕ_2 are isotopic. $\Rightarrow$ ϕ_2 is extendable.

$$\mathcal{M}(N_g) = \frac{\mathrm{Diff}(N_g)}{\text{isotopy}} \quad \text{the mapping class group of } N_g$$

For $\phi \in \mathcal{M}(N_g)$, ϕ preserves $q_{os} \overset{?}{\Rightarrow} \phi$ is os-extendable.

Generators for $\mathcal{M}(N_g)$

c : a simple closed curve in N_g.

c is an A-circle $\Leftrightarrow$ the regular neighborhood of c is an annulus.

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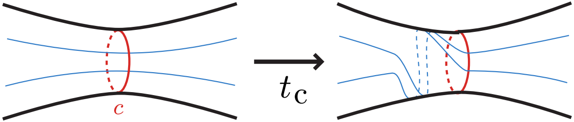

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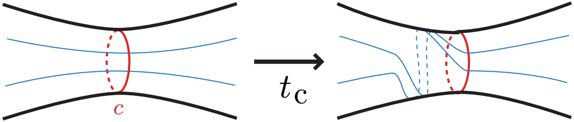

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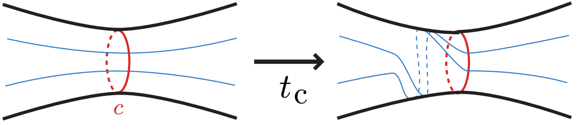

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Y-homeomorphisms are needed.

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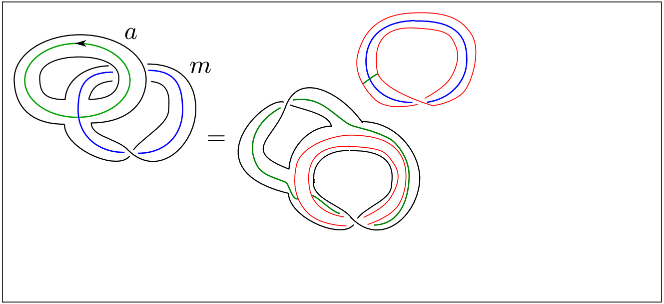

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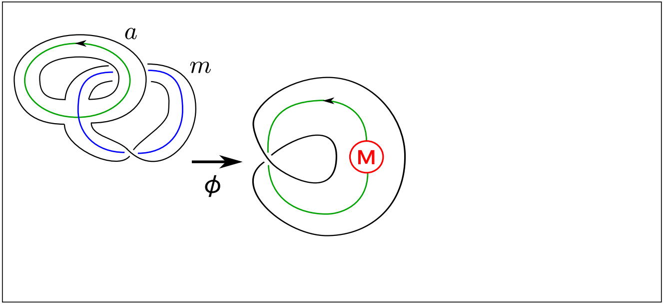

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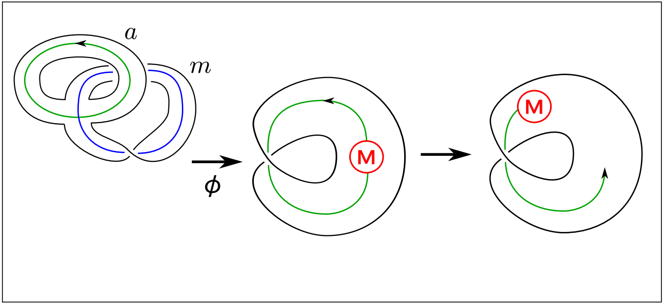

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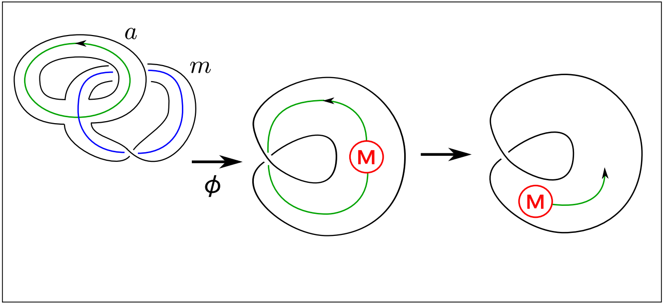

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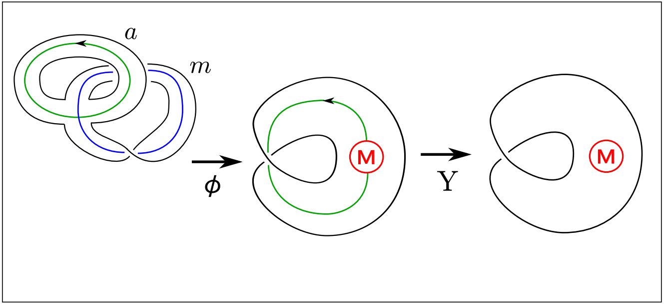

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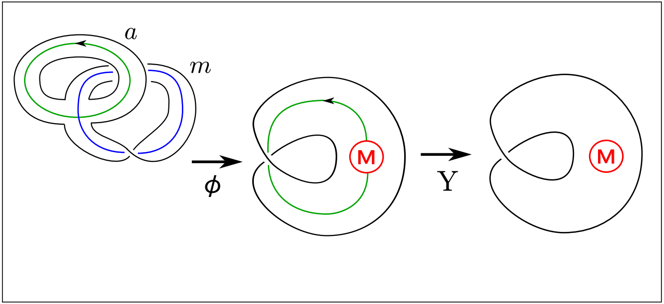

$$Y_{m,a}(x) = \begin{cases} \phi^{-1} \circ Y \circ \phi(x) & \text{if } x \text{ is in the neighborhood of } a \cup m, \\ x & \text{otherwise.} \end{cases}$$

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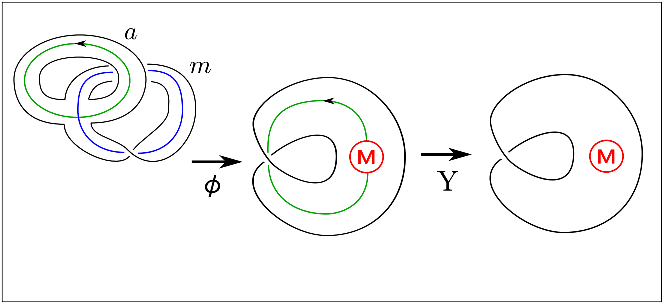

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Generators for $\mathcal{N}_g(q_{os})$

ϕ: a diffeomorphism over N_g.

$\phi_* : H_1(N_g;\mathbb{Z}_2) \to H_1(N_g;\mathbb{Z}_2)$: the homomorphism induced by ϕ.

$$\mathcal{N}_g(q_{os}) := \left\{ \phi \in \mathcal{M}(N_g) \;\middle|\; \begin{array}{l} q_{os}(\phi_*(x)) = q_{os}(x) \\ \text{for any } x \in H_1(N_g;\mathbb{Z}_2) \end{array} \right\},$$

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$$\mathcal{O}_g(q_{os}) := \{\phi_* \in Aut(H_1(N_g;\mathbb{Z}_2), <>_2) \,|\, \phi \in \mathcal{N}_g(q_{os})\}.$$

$$0 \to \Gamma_2(N_g) \to \mathcal{N}_g(q_{os}) \to \mathcal{O}_g(q_{os}) \to 0$$

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$$\Gamma_2(N_g) := \{\phi \in \mathcal{M}(N_g) \,|\, \phi_* = id_{H_1(N_g;\mathbb{Z}_2)}\}$$

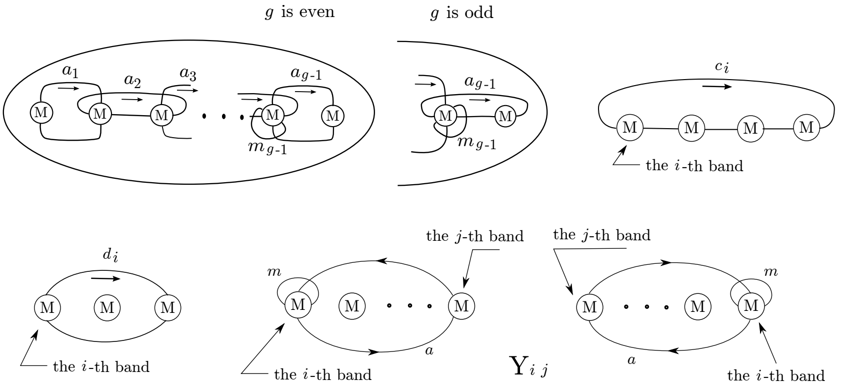
$$\mathcal{O}_g(q_{os}) := \{\phi_* \in Aut(H_1(N_g;\mathbb{Z}_2), <>_2) \,|\, \phi \in \mathcal{N}_g(q_{os})\}.$$

$$0 \to \Gamma_2(N_g) \to \mathcal{N}_g(q_{os}) \to \mathcal{O}_g(q_{os}) \to 0$$

By using
Szepietowski's generators for $\Gamma_2(N_g)$ (arXiv:1108.3927)
Nowik's generators for $\mathcal{O}_g(q_{os})$ (Top. and its appli. 154 (2007))
We find a finite system of generators for $\mathcal{N}_g(q_{os})$:

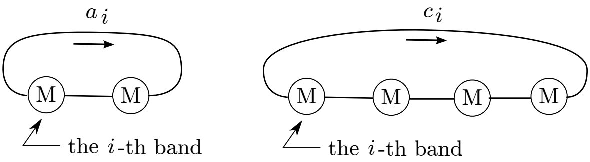
Lemma

$\mathcal{N}_g(q_{os})$ is generated by $t_{a_1}^2, \ldots, t_{a_{g-1}}^2, t_{c_1}^2, \ldots, t_{c_{g-3}}^2, t_{d_1}, \ldots, t_{d_{g-2}}$, $t_{a_1}t_{a_3}t_{c_1}, \ldots, t_{a_{g-3}}t_{a_{g-1}}t_{c_{g-3}}$, and $Y_{i,j}$ $(i, j = 1, \ldots g, i \neq j)$.



Lemma

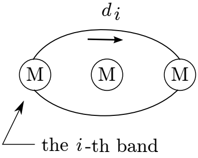
$\mathcal{N}_g(q_{os})$ is generated by $t^2_{a_1}$, ..., $t^2_{a_{g-1}}$, $t^2_{c_1}$, ..., $t^2_{c_{g-3}}$, t_{d_1}, ..., $t_{d_{g-2}}$, $t_{a_1}t_{a_3}t_{c_1}$, ..., $t_{a_{g-3}}t_{a_{g-1}}t_{c_{g-3}}$, and $Y_{i,j}$ $(i,j = 1,\dots g, i \neq j)$.



The regular neighborhoods of a_i and c_i are flat annuli whose core is trivial knot in $S^3 \times \{0\} \Rightarrow t^2_{a_i}$ and $t^2_{c_i}$ are os-extendable by the same argumet in the beginning of this talk.

Lemma

$\mathcal{N}_g(q_{os})$ is generated by $t_{a_1}^2, \ldots, t_{a_{g-1}}^2, t_{c_1}^2, \ldots, t_{c_{g-3}}^2, t_{d_1}, \ldots, t_{d_{g-2}}$, $t_{a_1}t_{a_3}t_{c_1}, \ldots, t_{a_{g-3}}t_{a_{g-1}}t_{c_{g-3}}$, and $Y_{i,j}$ $(i, j = 1, \ldots g, i \neq j)$.

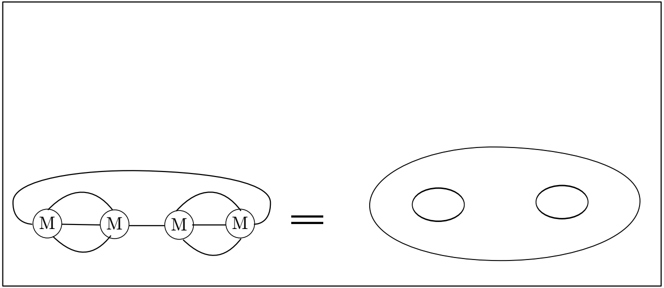


The regular neighborhoods of d_i is a Hopf band.
Hopf link ($= \partial$ (the Hopf band)) is the fiber link, whose fiber is the Hopf band and whose monodromy is t_{d_i}.
$\Rightarrow t_{d_i}$ is os-extendable.

Lemma

$\mathcal{N}_g(q_{os})$ is generated by $t_{a_1}^2$, …, $t_{a_{g-1}}^2$, $t_{c_1}^2$, …, $t_{c_{g-3}}^2$, t_{d_1}, …, $t_{d_{g-2}}$, $t_{a_1}t_{a_3}t_{c_1}$, …, $t_{a_{g-3}}t_{a_{g-1}}t_{c_{g-3}}$, and $Y_{i,j}$ $(i, j = 1, \ldots g, i \neq j)$.

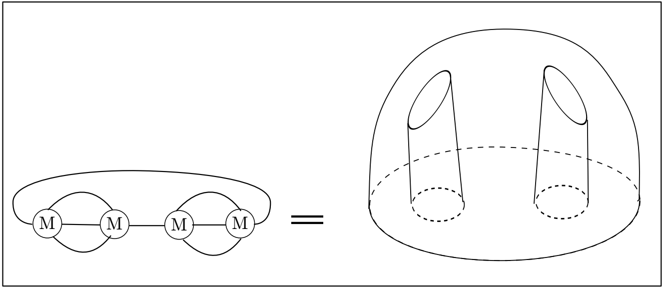
$t_{a_i}t_{a_{i+2}}t_{c_i}$ is os-extendable.



Lemma

$\mathcal{N}_g(q_{os})$ is generated by $t_{a_1}^2, \ldots, t_{a_{g-1}}^2, t_{c_1}^2, \ldots, t_{c_{g-3}}^2, t_{d_1}, \ldots, t_{d_{g-2}}$, $t_{a_1}t_{a_3}t_{c_1}, \ldots, t_{a_{g-3}}t_{a_{g-1}}t_{c_{g-3}}$, and $Y_{i,j}$ $(i, j = 1, \ldots g, i \neq j)$.

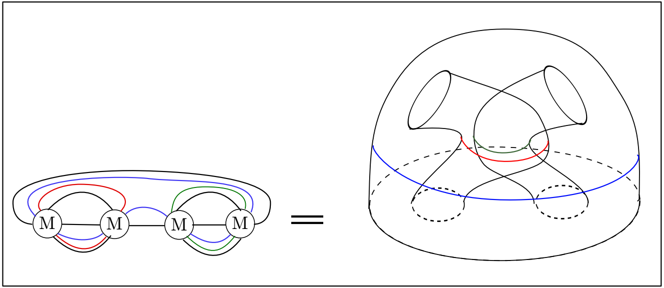
$t_{a_i}t_{a_{i+2}}t_{c_i}$ is os-extendable.



Lemma

$\mathcal{N}_g(q_{os})$ is generated by $t_{a_1}^2$, …, $t_{a_{g-1}}^2$, $t_{c_1}^2$, …, $t_{c_{g-3}}^2$, t_{d_1}, …, $t_{d_{g-2}}$, $t_{a_1}t_{a_3}t_{c_1}$, …, $t_{a_{g-3}}t_{a_{g-1}}t_{c_{g-3}}$, and $Y_{i,j}$ $(i, j = 1, \dots g, i \neq j)$.

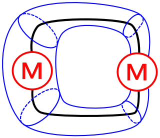
$t_{a_i}t_{a_{i+2}}t_{c_i}$ is os-extendable.



Lemma

$\mathcal{N}_g(q_{os})$ is generated by $t_{a_1}^2, \ldots, t_{a_{g-1}}^2, t_{c_1}^2, \ldots, t_{c_{g-3}}^2, t_{d_1}, \ldots, t_{d_{g-2}}$, $t_{a_1}t_{a_3}t_{c_1}, \ldots, t_{a_{g-3}}t_{a_{g-1}}t_{c_{g-3}}$, and $Y_{i,j}$ $(i, j = 1, \ldots g, i \neq j)$.

$Y_{i,j}$ are os-extendable by sliding the Möbius band along the tube shown in the following figure.



Lemma

$\mathcal{N}_g(q_{os})$ is generated by $t_{a_1}^2$, …, $t_{a_{g-1}}^2$, $t_{c_1}^2$, …, $t_{c_{g-3}}^2$, t_{d_1}, …, $t_{d_{g-2}}$, $t_{a_1}t_{a_3}t_{c_1}$, …, $t_{a_{g-3}}t_{a_{g-1}}t_{c_{g-3}}$, and $Y_{i,j}$ $(i,j = 1, \ldots g, i \neq j)$.

Every generators of $\mathcal{N}_g(q_{os})$ are os-extendable!!

Lemma

$\mathcal{N}_g(q_{os})$ is generated by $t_{a_1}^2, \ldots, t_{a_{g-1}}^2, t_{c_1}^2, \ldots, t_{c_{g-3}}^2, t_{d_1}, \ldots, t_{d_{g-2}}$, $t_{a_1}t_{a_3}t_{c_1}, \ldots, t_{a_{g-3}}t_{a_{g-1}}t_{c_{g-3}}$, and $Y_{i,j}$ $(i, j = 1, \ldots g, i \neq j)$.

Every generators of $\mathcal{N}_g(q_{os})$ are os-extendable!!
We see:

Main Theorem [H. (arXiv:1109.1668)]

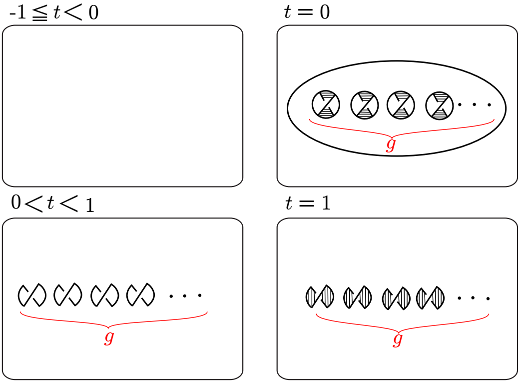
A diffeomorphism ϕ over the N_g is os-extentable to S^4
$\Leftrightarrow$ ϕ preserves the Guillou-Marin quadratic form q_{os}.

Problem

How about other stadard embedding of N_g into S^4.

Example: The embedding $N_g \overset{ps}{\hookrightarrow} S^4$ is p(parallel)-standard $\underset{Def}{\Longleftrightarrow}$

Under the decomposition $S^4 = D^4_- \cup S^3 \times [-1,1] \cup D^4_+$, the image of ps is in $S^3 \times [-1,1]$ and as shown in the following figure.



Thank you very much!!!

SHONO, Shounsai:
Flower basket, "Doto (surging waves)" 1956
The National Museum of Modern Art, Tokyo