

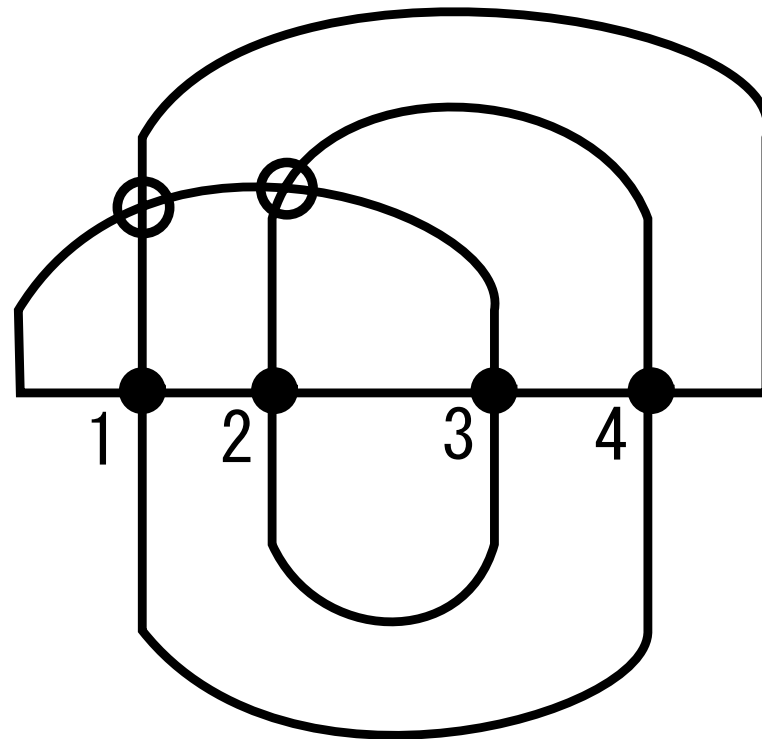
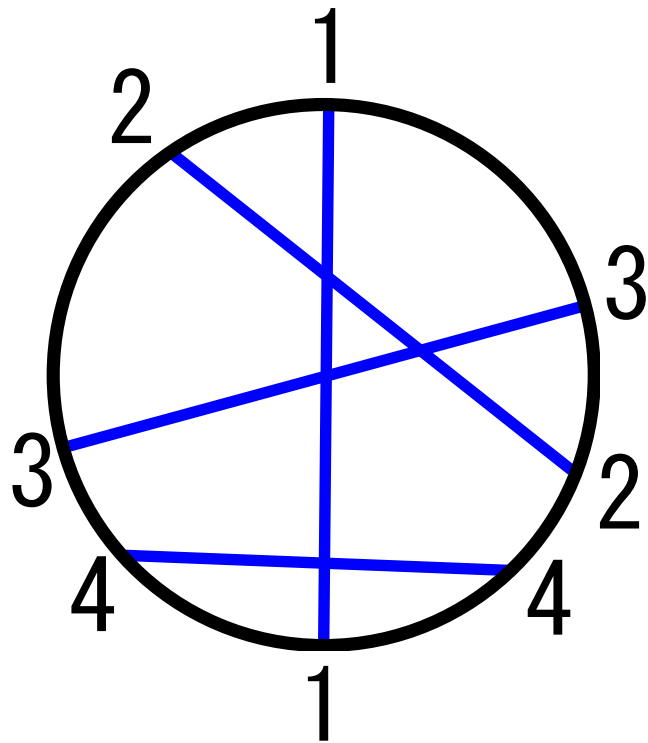
Jan, 11, 2012

On free knots

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Chord diagram

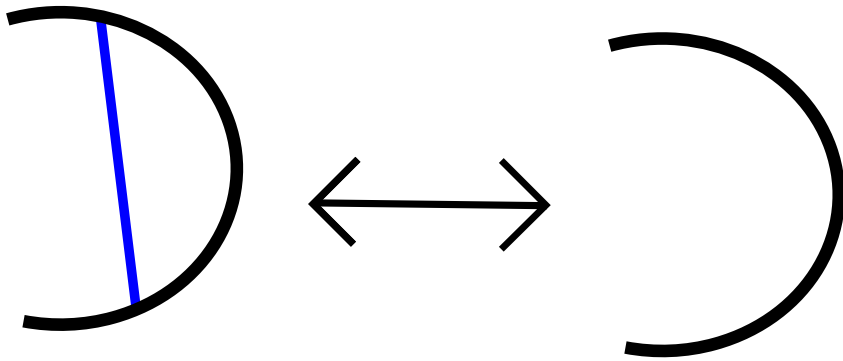
A *chord diagram* is a circle with several chords.



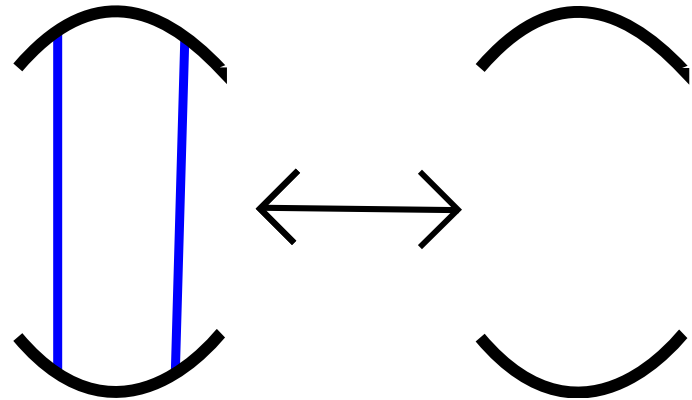
Free knot

$\{\textit{free knot}\} \stackrel{\text{def}}{=} \{\text{chord diagram}\} / (\text{R1}), (\text{R2}) \text{ and } (\text{R3}).$

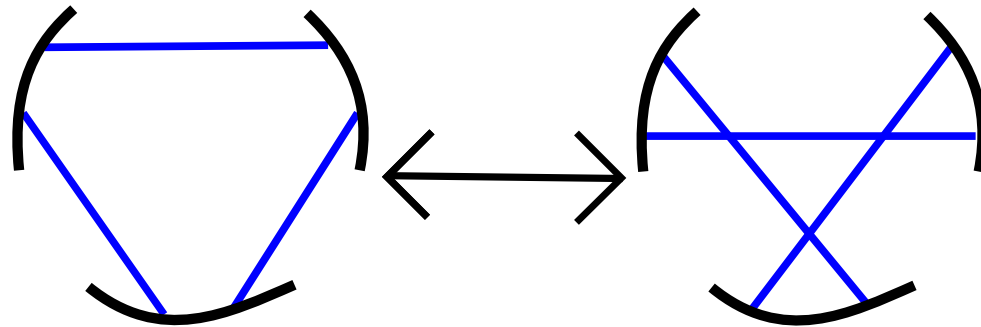
(R1)



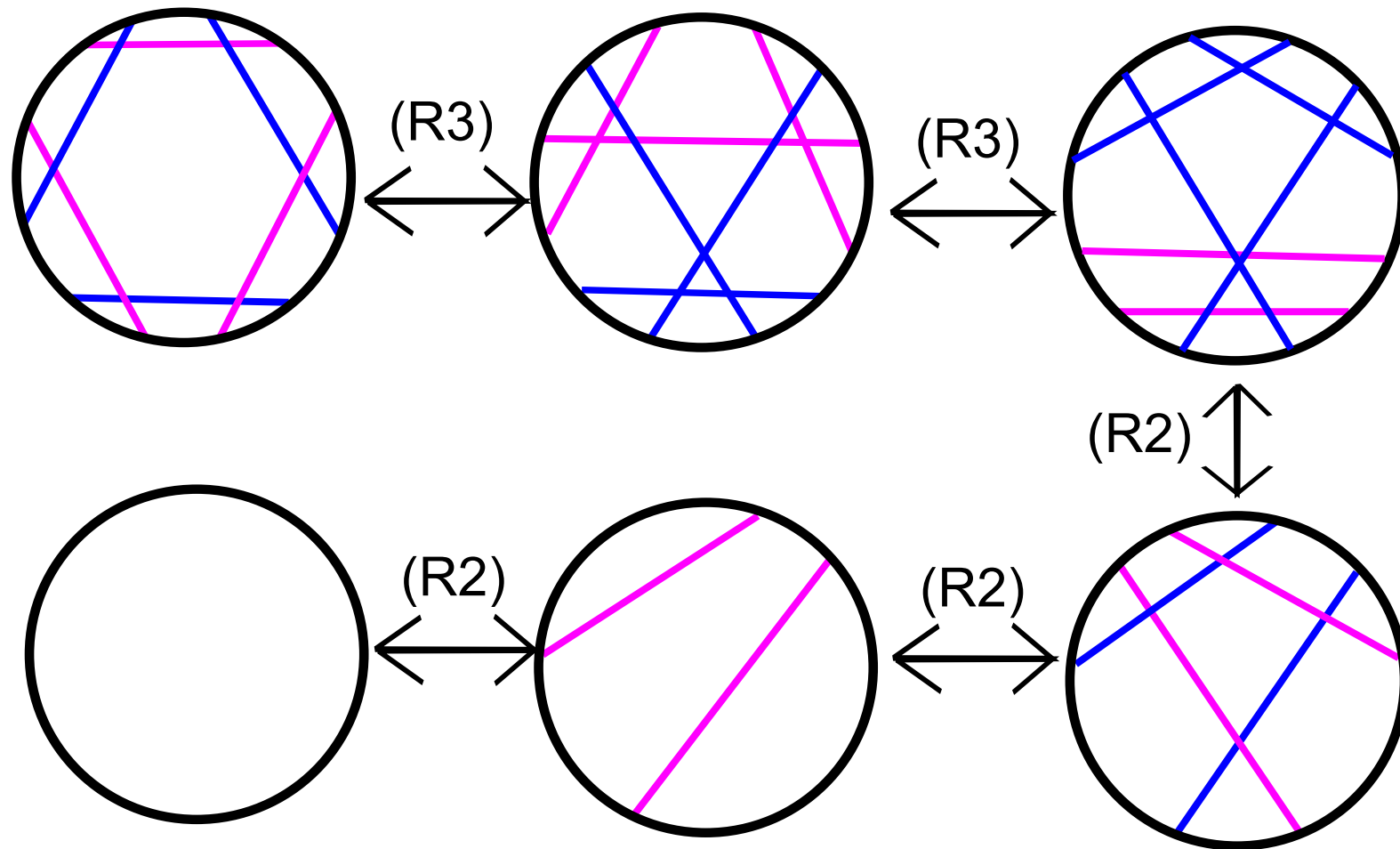
(R2)



(R3)



Example



Turaev conjectured any free knot was trivial.
Manturov and Gibson answered negatively.

Def

D : chord diagram.

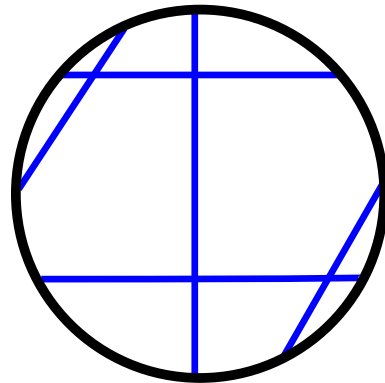
$c(D)$: the number of chords of D .

K : free knot.

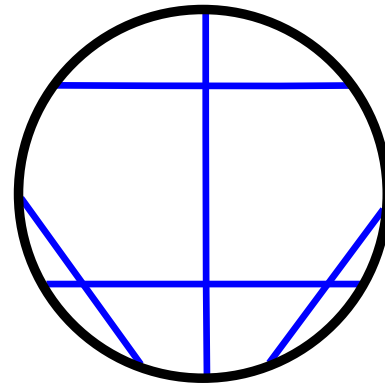
$c(K) \stackrel{\text{def}}{=} \min\{c(D) \mid D : \text{chord diagram of } K\}$.

Prop

- $c(K)=0 \Leftrightarrow K$ is a trivial free knot 0.1.
- $\nexists K$ s.t. $c(K)=1, 2, 3$ or 4.
- $c(K)=5 \Leftrightarrow K$:



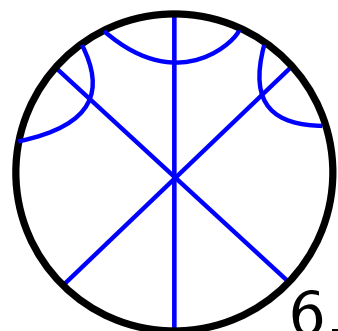
5.1



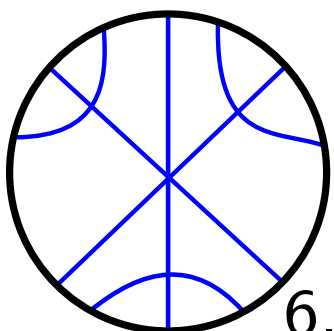
5.2

5.1 and 5.2 are mutually distinct.

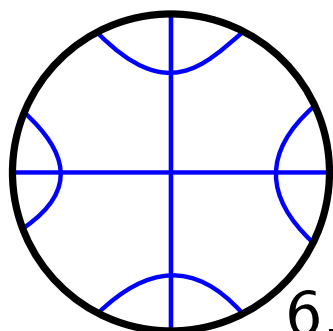
• $c(K)=6 \Rightarrow K: 6.1, \dots, 6.12, T_1 \text{ or } T_2.$



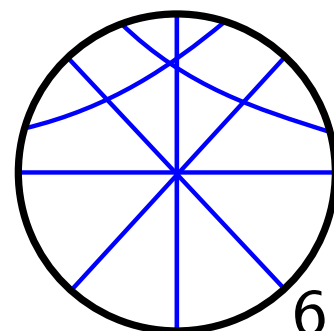
6.1



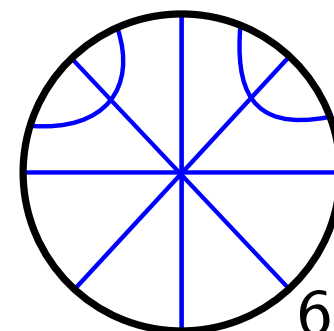
6.2



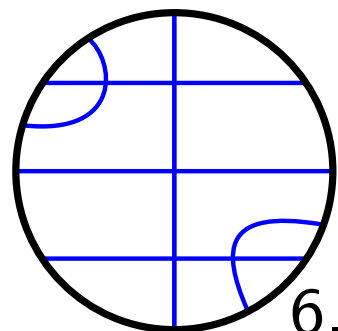
6.3



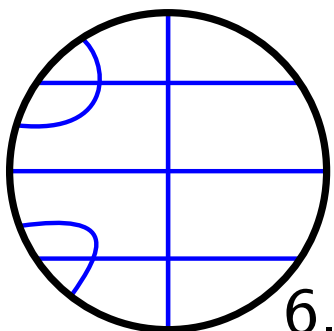
6.4



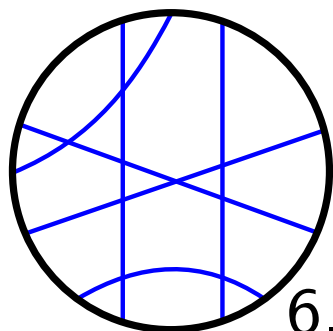
6.5



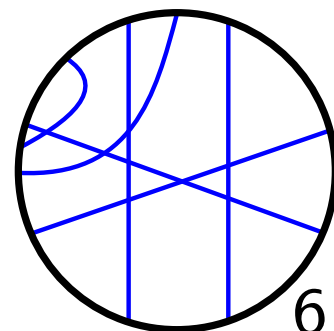
6.6



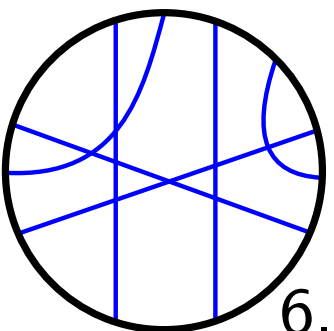
6.7



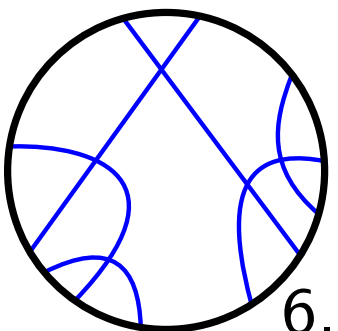
6.8



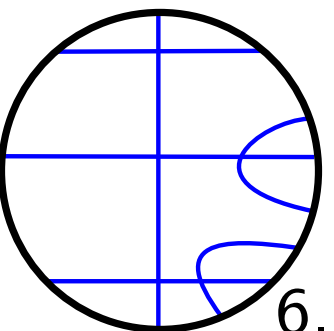
6.9



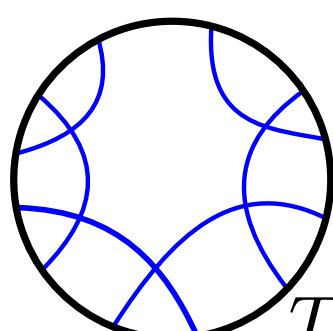
6.10



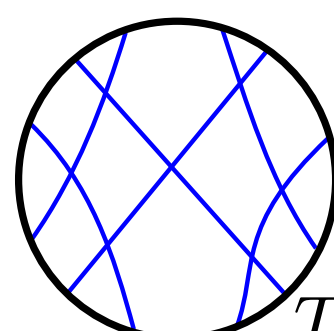
6.11



6.12



T_1



T_2

6.1, ..., 6.11 and 6.12 are mutually distinct with $c(K)=6$.
 $c(T_1), c(T_2)=0$ or 6 .

Invariants of free knots

1. $L(K) \in \mathbb{Z}_{\geq 0}$ (Manturov) .
2. $\{K\} \in \mathbb{Z}_2\mathbb{G}$ (Manturov) .
3. $\Delta(K) \in \mathbb{Z}_2\mathbb{K}_2$ (Manturov, Gibson) .

Prop [Manturov]

$\forall K$: free knot, $L(K) \equiv 0 \pmod{4}$.

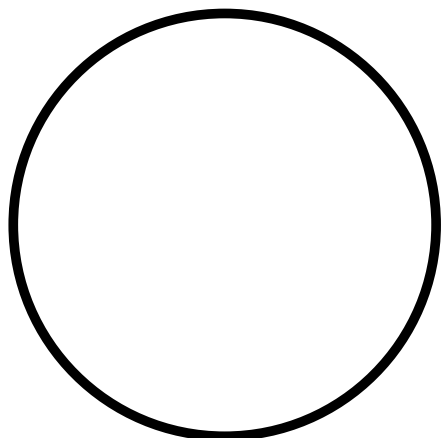
Conj [Manturov]

$\forall K$: free knot, $L(K) \equiv 0 \pmod{8}$.

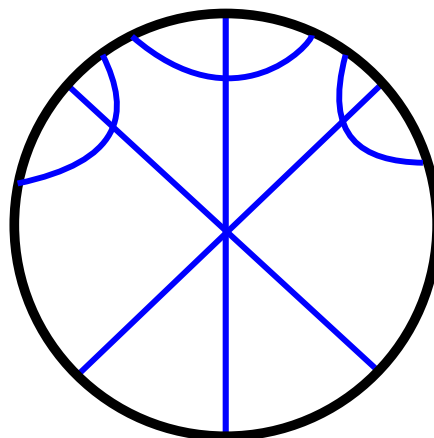
Thm

$\forall K: \text{free knot}, L(K) \equiv 0 \pmod{8} .$

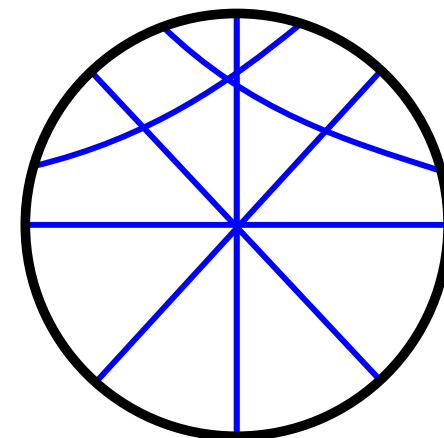
Example



$$L(0.1)=0$$

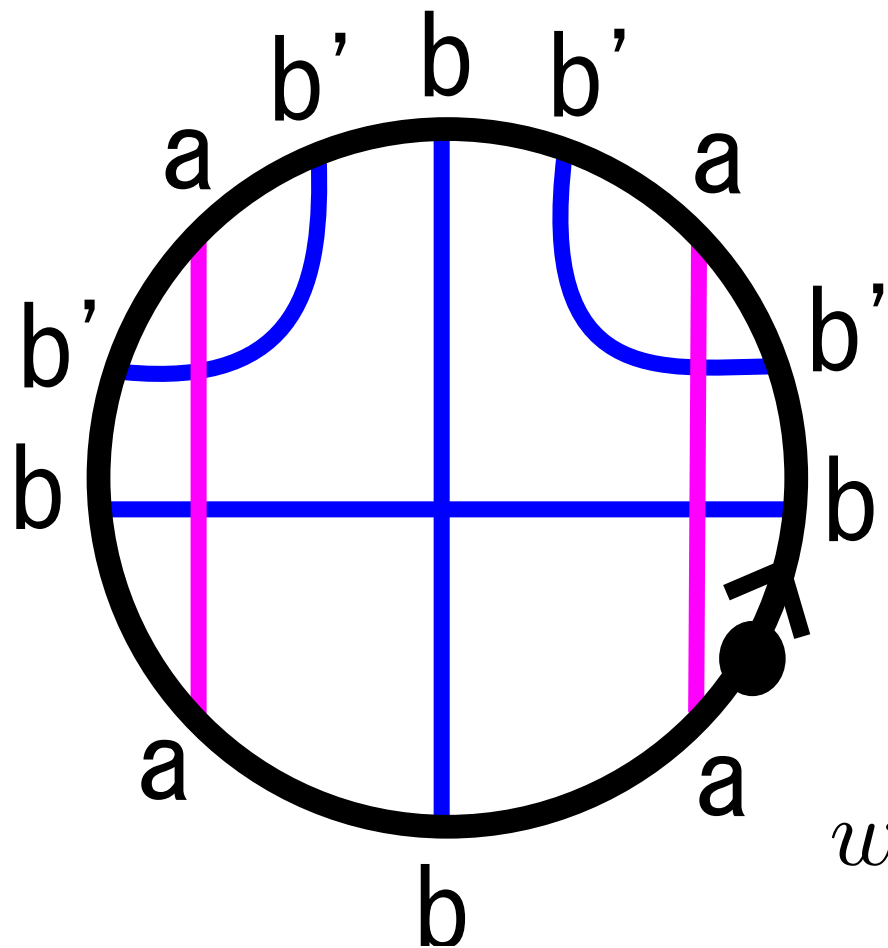


$$L(6.1)=0$$



$$L(6.4)=8$$

$c(K) \leq 6, L(K)=8 \Leftrightarrow K: 5.2, 6.4, 6.5, 6.6 \text{ or } 6.7.$



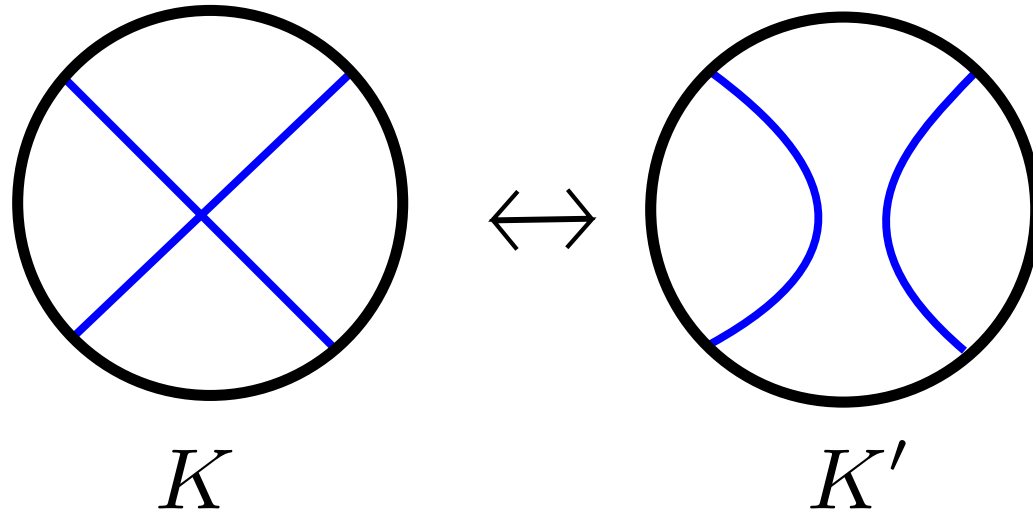
$$w = b b' a b' b b' a b' b a b a.$$

$$\exists n \in \mathbb{Z}; w = (b b')^n \in \left\langle a, b, b' \mid \begin{array}{l} a^2 = b^2 = b'^2 = 1 \\ ab = b'a \end{array} \right\rangle$$

$$L(K) := 2|n|.$$

Lem

K, K' : free knots s.t.



$$\Rightarrow L(K) \equiv L(K') \pmod{8} .$$