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Epimorphisms between knot groups and special values of colored Jones polynomials

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1 Definition

This is joint work with Teruaki KITANO.

- $K \subset S^3$: a knot in S^3
- $G(K) = \pi_1(S^3 - K)$: a knot group of K

If an epimorphism exists between two knot groups $G(K_1)$, $G(K_2)$ and it preserves meridian, we can define a partial order $K_1 \geq K_2$.

2 Partial order of prime knot

Until now, the partial order is determined on the set of prime knots with up to 11-crossings.

- In 2005 : up to 10 crossings (Kitano-Suzuki)

- In 2010 : up to 11 crossings (Horie-Kitano-Matsumoto-Suzuki)

The list with up to 11-crossings of the partial order are the following.

Theorem (Horie-Kitano-Matsumoto-Suzuki)

$$\left.\begin{array}{l} 8_5, 8_{10}, 8_{15}, 8_{18}, 8_{19}, 8_{20}, 8_{21}, \\ 9_1, 9_6, 9_{16}, 9_{23}, 9_{24}, 9_{28}, 9_{40}, \\ 10_5, 10_9, 10_{32}, 10_{40}, 10_{61}, 10_{62}, \\ 10_{63}, 10_{64}, 10_{65}, 10_{66}, 10_{76}, 10_{77}, 10_{78}, \\ 10_{82}, 10_{84}, 10_{85}, 10_{87}, 10_{98}, \\ 10_{99}, 10_{103}, 10_{106}, 10_{112}, 10_{114}, \\ 10_{139}, 10_{140}, 10_{141}, 10_{142}, 10_{143}, \\ 10_{144}, 10_{159}, 10_{164}, \end{array}\right\} \geq 3_1$$

$$\left.\begin{array}{l}
11a_{43}, 11a_{44}, 11a_{46}, 11a_{47}, 11a_{57}, 11a_{58},\\
11a_{71}, 11a_{72}, 11a_{73}, 11a_{100}, 11a_{106}, 11a_{107},\\
11a_{108}, 11a_{109}, 11a_{117}, 11a_{134}, 11a_{139},\\
11a_{157}, 11a_{165}, 11a_{171}, 11a_{175}, 11a_{176},\\
11a_{194}, 11a_{196}, 11a_{203}, 11a_{212}, 11a_{216},\\
11a_{223}, 11a_{231}, 11a_{232}, 11a_{236}, 11a_{244},\\
11a_{245}, 11a_{261}, 11a_{263}, 11a_{264}, 11a_{286},\\
11a_{305}, 11a_{306}, 11a_{318}, 11a_{332}, 11a_{338},\\
11a_{340}, 11a_{351}, 11a_{352}, 11a_{355},\\
11n_{71}, 11n_{72}, 11n_{73}, 11n_{74}, 11n_{75}, 11n_{76}, 11n_{77},\\
11n_{78}, 11n_{81}, 11n_{85}, 11n_{86}, 11n_{87}, 11n_{94},\\
11n_{104}, 11n_{105}, 11n_{106}, 11n_{107}, 11n_{136},\\
11n_{164}, 11n_{183}, 11n_{184}, 11n_{185},
\end{array}\right\} \geq 3_1$$

$$\left.\begin{array}{l} 9_{18}, 9_{37}, 9_{40}, 9_{58}, 9_{59}, 9_{60}, \\ 10_{122}, 10_{136}, 10_{137}, 10_{138}, \\ 11a_{5}, 11a_{6}, 11a_{51}, 11a_{132}, 11a_{239}, 11a_{297}, \\ 11a_{348}, 11a_{349}, 11n_{100}, 11n_{148}, 11n_{157}, 11n_{165} \end{array}\right\} \geq 4_1$$

$$11n_{78}, 11n_{148} \geq 5_1$$

$$10_{74}, 10_{120}, 10_{122}, 11n_{71}, 11n_{185} \geq 5_2$$

$$11a_{352} \geq 6_1$$

$$11a_{351} \geq 6_2$$

$$11a_{47}, 11a_{239} \geq 6_3$$

3 Colored Jones polynomial

Colored Jones polynomial $J_{K,N}(t)$ is a Laurent polynomial of variable t. It can be defined by using $N+1$-dimensional representation of quantum group $U_t(sl_2\mathbb{C})$.

The following equality is volume conjecture by Kashaev, Murakami and Murakami.

Volume conjecture (Kashaev-Murakami-Murakami)

$$\lim_{N\to\infty} \frac{\log|J_{K,N}(\xi_{N+1})|}{N} = \frac{1}{2\pi}||S^3 - K||$$

$||S^3 - K||$: The Gromov's simplicial volume.

ξ_{N+1}=exp$(\frac{2\pi\sqrt{-1}}{N+1})$: $N+1$-th root of a unity.

We conjecture relation of a partial order and special values of the colored Jones polynomial.

Conjecture

$$K_1 \geq K_2 \Rightarrow |J_{K_1,N}(\xi_{N+1})| \geq |J_{K_2,N}(\xi_{N+1})|$$

N is a sufficiently large integer determined by K_1 and K_2.

When a partial order $K_1 \geq K_2$ holds, we compute special values of the colored Jones polynomial for K_1 and K_2. Further, we check whether inequality holds.

4 Computation

We compute special values of colored Jones polynomial by Mathematica. But, it is hard to compute special values of colored Jones polynomial. We try to compute using two ways.

- Knot Theory (Bar-Natan)
- Colored Jones polynomial for a torus knot (H. R. Morton)

First, we compute by Knot Theory.

Example1

We consider the following partial order.

$$\left.\begin{array}{l} 8_5, 8_{10}, 8_{15}, 8_{18}, 8_{19}, 8_{20}, 8_{21}, \\ 9_1, 9_6, 9_{16}, 9_{23}, 9_{24}, 9_{28}, 9_{40} \end{array}\right\} \geq 3_1$$

We made computation by computer for $N = 1, 2, 3, 4, 5, 6$. These inequality of special values of colored Jones polynomial holds. However for $N = 7$, the inequality does not hold for $9_1 \geq 3_1$,

$$|J_{9_1,7}(\xi_8)| < |J_{3_1,7}(\xi_8)|$$

Example2

We consider the following partial order.

$$\left.\begin{array}{l} 9_{18}, 9_{37}, 9_{40}, 10_{59}, 10_{60}, \\ 10_{122}, 10_{136}, 10_{137}, 10_{138} \end{array}\right\} \geq 4_1$$

We made computation by computer for $N = 1, 2, 3, 4, 5$. These inequality of special values of colored Jones polynomial holds. However for $N = 2$, the inequality does not hold for $10_{136} \geq 4_1$,

$$|J_{10_{136},2}(\xi_3)| < |J_{4_1,2}(\xi_3)|$$

We compute colored Jones polynomial for large N. But, Mathematica stops computing, because the calculation of colored Jones polynomial is complicated.
Knot Theory exactly computes colored Jones polynomial. But we need special values of colored Jones polynomial.

Here, we consider the colored Jones polynomial by the Morton's formula for a torus knot.

Theorem (H. R. Morton)

$$J_N(T(a,b);t) =$$

$$\frac{t^{-ab(N^2-1)/4}}{t^{N/2}-t^{-N/2}} \sum_{\varepsilon=\pm 1} \sum_{k=-(N-1/2)}^{(N-1)/2} \varepsilon t^{abk^2+k(a+\varepsilon b)+\varepsilon/2}$$

The following knots can be represented as a torus knot.

$3_1 = T(3,2)$, $5_1 = T(5,2)$, $7_1 = T(7,2)$,
$8_{19} = T(4,3)$, $9_1 = T(9,2)$, $11a_{367} = T(11,2)$

There are three torus knots in the list of partial order.

$$T(3,2) = 3_1,\ T(3,4) = 8_{19},\ T(9,2) = \ 9_1$$

We compute special values of the colored Jones polynomial for these torus knots.

The following data is the difference between special values of the colored Jones polynomial for K_1 and K_2.

$8_{19} \geq 3_1$			
N	Difference	N	Difference
2	0	24	0.044360
3	0.054535	25	0.025058
4	0.161158	26	0.037776
5	0.088756	27	0.018627
6	0.157511	28	0.025507
7	0.086361	29	0.008962
8	0.131463	30	0.003434
9	0.070143	31	-0.000948
10	0.099250	32	-0.078622
11	0.047561	33	-0.000787
12	0.061402	34	0.003228
13	0.021348	35	0.007678
14	0.009084	36	0.020116
15	-0.001804	37	0.013901
16	-0.146125	38	0.026130
17	-0.001074	39	0.016324
18	0.007275	40	0.026818
19	0.014851	41	0.015549
20	0.036619	42	0.023653
21	0.024760	43	0.012014
22	0.045064	44	0.016494
23	0.027502	45	0.006014

$9_1 \geq 3_1$			
N	Difference	N	Difference
2	0.549306	24	0.046398
3	0.345900	25	0.046479
4	0.191202	26	-0.002527
5	0.153080	27	-0.099999
6	0.179264	28	-0.002457
7	0.162932	29	0.040159
8	-0.012216	30	0.037113
9	-0.288395	31	0.026574
10	-0.006739	32	0.025830
11	0.106211	33	0.033802
12	0.093909	34	0.034354
13	0.065006	35	-0.001533
14	0.060389	36	-0.076551
15	0.075682	37	-0.001476
16	0.073982	38	0.030886
17	-0.003206	39	0.028743
18	-0.144474	40	0.020761
19	-0.003087	41	0.020264
20	0.058746	42	0.026642
21	0.053427	43	0.027220
22	0.037710	44	-0.001216
23	0.036016	45	-0.061836

$8_{19} \geq 3_1$			
N	Difference	N	Difference
46	0.002472	74	0.013358
47	-0.000629	75	0.006811
48	-0.050874	76	0.009490
49	-0.000694	77	0.003457
50	0.002163	78	0.001406
51	0.005175	79	-0.000364
52	0.013869	80	-0.030903
53	0.009646	81	-0.000348
54	0.018319	82	0.001353
55	0.011528	83	0.003229
56	0.019098	84	0.008615
57	0.011144	85	0.006041
58	0.017094	86	0.011526
59	0.008719	87	0.007300
60	0.012084	88	0.012166
61	0.004403	89	0.007139
62	0.001820	90	0.011016
63	-0.000460	91	0.005652
64	-0.038180	92	0.007872
65	-0.000414	93	0.002890
66	0.001735	94	0.001194
67	0.004033	95	-0.000310
68	0.010655	96	-0.025514
69	0.007449	97	-0.000319
70	0.014146	98	0.001126
71	0.008923	99	0.002691
72	0.014836	100	0.007226
73	0.008662	101	0.005075

$9_1 \geq 3_1$			
N	Difference	N	Difference
46	-0.001220	74	0.015771
47	0.024847	75	0.014876
48	0.023220	76	0.010882
49	0.016828	77	0.010739
50	0.016486	78	0.014313
51	0.021839	79	0.014788
52	0.022431	80	-0.000748
53	-0.001105	81	-0.033590
54	-0.051380	82	-0.000717
55	-0.001037	83	0.014090
56	0.020850	84	0.013313
57	0.019582	85	0.009758
58	0.014275	86	0.009647
59	0.014035	87	0.012857
60	0.018640	88	0.013295
61	0.019198	89	-0.000647
62	-0.000905	90	-0.030275
63	-0.043591	91	-0.000642
64	-0.000858	92	0.012712
65	0.018001	93	0.012016
66	0.016943	94	0.008806
67	0.012378	95	0.008714
68	0.012181	96	0.011629
69	0.016197	97	0.012038
70	0.016701	98	-0.000600
71	-0.000844	99	-0.027658
72	-0.037906	100	-0.000594
73	-0.000816	101	0.011567

$8_{19} \geq 3_1$			
N	Difference	N	Difference
102	0.009706	...	....
103	0.006158	411	0.001248
104	0.010284	412	0.001755
105	0.006045	413	0.000648
106	0.009348	414	0.000269
107	0.004801	415	-0.000070
108	0.006704	416	-0.005906
109	0.002460	417	-0.000069
110	0.001018	418	0.000266
111	-0.000261	419	-----
112	-0.021886	420	0.001722
113	-0.000248	421	-----
114	0.000990	422	0.002347
115	0.002338	423	-----
116	0.006240	424	0.002522
117	0.004389	:	:
118	0.008392	489	-----
119	0.005327	490	0.002021
120	0.008907	491	-----
121	0.005234	492	0.001470
122	0.008110	493	-----
123	0.004163	494	0.000225
124	0.005826	495	-----
125	0.002137	496	-0.004950
126	0.000878	497	-----
127	0.012014	498	0.000224
128	-0.019250	499	-----
:	:	500	0.001447

$9_1 \geq 3_1$			
N	Difference	N	Difference
102	0.010946	:	:
103	0.008031	335	0.003488
104	0.007958	336	0.003324
105	0.010636	337	0.002456
106	0.011026	338	0.002448
107	-0.000531	339	0.003294
108	-0.025437	340	0.003437
109	-0.000523	341	-0.000170
110	0.010635	342	-----
111	0.010071	343	-----
112	0.007396	344	-----
113	0.007331	345	-----
114	0.009802	346	-----
115	0.010167	347	-----
116	-0.000490	348	-----
117	-0.023521	:	:
118	-0.000486	489	-----
119	0.009819	490	-----
120	0.009303	491	-----
121	0.006835	492	-----
122	0.006778	493	-----
123	0.009075	494	-----
124	0.009421	495	-----
125	-0.000465	496	-----
126	-0.021821	497	-----
127	-0.000454	498	-----
128	0.009129	499	-----
:	:	500	-----

5 Period

For the previous list, if N becomes larger, the inequality often does not hold.
In the case of $8_{19} \geq 3_1$, the following N does not hold inequality.

	15	31	47	63	79	95	111	127	143	159	...
N	16	32	48	64	80	96	112	128	144	160	...
	17	33	49	65	81	97	113	129	145	161	...

...	207	223	239	255	271	287	303	319	335	...
...	208	224	240	256	272	288	304	320	336	...
...	209	225	241	257	273	289	305	321	337	...

By the above list, whenever N increased by 16, the inequality does not hold.

In the case of $9_1 \geq 3_1$, the following N does not hold inequality.

	8	17	26	35	44	53	62	71	80	...
N	9	18	27	36	45	54	63	72	81	...
	10	19	28	37	46	55	64	73	82	...

...	179	188	197	206	215	224	233	242	251	...
...	180	189	198	207	216	225	234	243	252	...
...	181	190	199	208	217	226	235	244	253	...

By the above list, whenever N increased by 9, the inequality does not hold.

From these results, we observe the period.

We check the periodicity.

$$\frac{log|J_N(T(3,2);(\xi_{N+1}))|}{N}$$

$$\frac{log|J_N(T(3,4);(\xi_{N+1}))|}{N}$$

$$\frac{log|J_N(T(9,2);(\xi_{N+1}))|}{N}$$

We draw the graph for these special values.

Graph1

Silver-Whitten proved the following theorem.

Theorem (Silver-Whitten)
Let K be a (a, b)-torus knot, and let K' be a nontrivial knot. The following statements are equivalent.

- $K \geq K'$
- K' is a (a', b')-torus knot, for some a', $b' > 2$, such that $a'|a$ and $b'|b$, or $a'|b$ and $b'|a$.

For example, we compute the special value for $T(3, 8) \geq T(3, 2)$, $T(9, 4) \geq T(9, 2)$.

$T(3,8) \geq T(3,2)$			
N	Difference	N	Difference
2	0	30	0.001654
3	0.382143	31	-0.000726
4	0.331220	32	-0.112995
5	0.275258	33	-0.000699
6	0.253307	34	0.001489
7	0.070980	35	0.034644
8	-0.217233	36	0.038724
9	0.054954	37	0.038938
10	0.118991	38	0.041401
11	0.113017	39	0.013872
12	0.120669	40	-0.041204
13	0.113028	41	0.012668
14	0.125840	42	0.028594
15	0.053210	43	0.028982
16	-0.080503	44	0.032790
17	0.046613	45	0.032396
18	0.097682	46	0.037961
19	0.077199	47	0.016541
20	0.072382	48	-0.027654
21	0.059372	49	0.015877
22	0.054471	50	0.034882
23	0.022214	51	0.028513
24	-0.068653	52	0.027652
25	0.021323	53	0.023403
26	0.060123	54	0.022124
27	0.052953	55	0.009326
28	0.049424	56	-0.029958
29	0.041532	57	0.009377

$T(3,20) \geq T(3,2)$			
N	Difference	N	Difference
2	0	30	-0.044999
3	0.669639	31	0.075755
4	0.559374	32	-0.020794
5	0.180022	33	0.050660
6	0.410152	34	0.060857
7	0.224499	35	0.030826
8	0.331923	36	0.065791
9	0.251783	37	0.063471
10	-0.151925	38	0.069748
11	0.207381	39	0.018505
12	0.196629	40	-0.073152
13	0.146322	41	0.017574
14	0.190844	42	0.063082
15	0.070207	43	0.054579
16	-0.046826	44	0.053796
17	0.139073	45	0.023950
18	0.124056	46	0.044965
19	0.025551	47	0.035561
20	-0.167657	48	-0.013775
21	0.022560	49	0.047959
22	0.107839	50	-0.026774
23	0.089736	51	0.046094
24	0.111543	52	0.044442
25	0.039905	53	0.020938
26	0.043052	54	0.020796
27	0.040916	55	0.018222
28	0.082392	56	0.047869
29	0.080950	57	0.036246

$T(3,8) \geq T(3,2)$			
N	Difference	N	Difference
58	0.027047	86	0.013938
59	0.024363	87	0.005937
60	0.023213	88	-0.018879
61	0.019896	89	0.006034
62	0.000874	90	0.017442
63	-0.000238	91	0.015797
64	-0.059670	92	0.015128
65	-0.000243	93	0.013027
66	0.000805	94	0.000543
67	0.018105	95	-0.000209
68	0.020472	96	-0.038831
69	0.020823	97	-0.000194
70	0.022403	98	0.000530
71	0.007524	99	0.012225
72	-0.023257	100	0.013901
73	0.0070339	101	0.014214
74	0.016154	102	0.015373
75	0.016553	103	0.005200
76	0.018937	104	-0.016008
77	0.018904	105	0.004897
78	0.022378	106	0.011297
79	0.009860	107	0.011623
80	-0.016582	108	0.013349
81	0.009617	109	0.013377
82	0.021303	110	0.015888
83	0.017563	111	0.007034
84	0.017166	112	-0.011781
85	0.014643	113	0.006908

$T(3,20) \geq T(3,2)$			
N	Difference	N	Difference
58	0.040909	86	0.029164
59	0.008035	87	0.018466
60	-0.054438	88	0.030456
61	0.007891	89	0.025631
62	0.035869	90	-0.016767
63	0.037358	91	0.025047
64	-0.011809	92	0.025567
65	0.016017	93	0.020336
66	0.040297	94	0.028276
67	0.028242	95	0.010933
68	0.034592	96	-0.007918
69	0.033025	97	0.024233
70	-0.021648	98	0.022663
71	0.032102	99	0.004833
72	0.037193	100	-0.032804
73	0.021975	101	0.004681
74	0.033858	102	0.023252
75	0.012771	103	0.020057
76	0.030285	104	0.025781
77	0.026970	105	0.009554
78	0.000726	106	0.010617
79	-0.000080	107	0.010399
80	-0.063069	108	0.021428
81	-0.000076	109	0.021597
82	0.000695	110	-0.012176
83	0.025041	111	0.021199
84	0.027430	112	-0.005902
85	0.011297	113	0.014805

$T(3,8) \geq T(3,2)$			
N	Difference	N	Difference
114	0.015329	:	:
115	0.012676	296	-0.005627
116	0.012425	297	————
117	0.010627	298	0.004017
118	0.010145	299	————
119	0.004319	:	:
120	-0.013877	414	0.000125
121	0.004425	415	————
122	0.012853	416	-0.009002
123	0.011673	417	————
124	0.011213	418	0.000124
125	0.009685	419	————
126	0.000413	420	————
127	-0.000145	421	————
128	-0.029165	422	————
129	-0.000147	:	:
130	0.000397	489	————
131	0.009250	490	————
132	0.010544	491	————
133	0.010807	492	————
134	0.011712	493	————
135	0.003974	494	————
136	-0.012231	495	————
137	0.003763	496	————
138	0.008680	497	————
139	0.008948	498	————
140	0.010294	499	————
:	:	500	————

$T(3,20) \geq (3,2)$			
N	Difference	N	Difference
114	0.018151	:	:
115	0.009367	188	0.012307
116	0.020395	189	————
117	0.020041	190	-0.007070
118	0.022428	191	————
119	0.006018	:	:
120	-0.024665	260	-0.012674
121	0.005919	261	————
122	0.021688	262	0.009055
123	0.019058	263	————
124	0.019072	264	0.010156
125	0.008609	265	————
126	0.016413	266	————
127	0.013165	267	————
128	-0.005185	268	————
129	0.018233	:	:
130	-0.010338	489	————
131	0.017966	490	————
132	0.017531	491	————
133	0.008366	492	————
134	0.008404	493	————
135	0.007434	494	————
136	0.019721	495	————
137	0.015087	496	————
138	0.017195	497	————
139	0.003405	498	————
140	-0.023650	499	————
:	:	500	————

Graph2

In the case of $T(3,20) \geq T(3,2)$, the period is not a equal interval.

For following N, inequality does not hold.

10,	16,	20,	30,	32,	40,	48,	50,	60,	64,	70,	80,	
90,	96,	100,	110,	112,	120,	128,	130,	140,	144,	150,	160,	
170,	176,	180,	190,	192,	200,	208,	210,	220,	224,	230,	240,	...

In this case, the period appears to take the difference of adjacent N.
Difference of adjacent N are, (6, 4, 10, 2, 8, 8, 2, 10, 4, 6, 10, 10), (6, 4, 10, 2, 8, 8, 2, 10, 4, 6, 10, 10), (........)

$T(a', b') \geq T(a, b)$	Period
$T(3,4) \geq T(3,2)$	Every 16
$T(3,8) \geq T(3,2)$	Every 8
$T(3,10) \geq T(3,2)$	Every10
$T(3,14) \geq T(3,2)$	Every 14
$T(3,16) \geq T(3,2)$	Every 8
$T(3,22) \geq T(3,2)$	Every 11
$T(3,26) \geq T(3,2)$	Every 13
$T(9,2) \geq T(3,2)$	Every 9
$T(9,8) \geq T(3,2)$	Every 32
$T(9,10) \geq T(3,2)$	Every 40
$T(3,20) \geq T(3,4)$	Every 10
$T(45,4) \geq T(15,2)$	Every 9
$T(9,16) \geq T(9,2)$	Every 16
$T(9,20) \geq T(9,2)$	Every 20

We computed $T(3,5)$, $T(3,2)$ and $T(9,5)$, $T(3,2)$. These patterns are not partial order. But, the period can be observed.

$T(3,5), T(3,2)$			
N	Difference	N	Difference
2	-0.549306	22	-0.044513
3	0.242726	23	0.035385
4	0.171995	24	0.031910
5	-0.091333	25	-0.014793
6	-0.048142	26	-0.008639
7	0.135690	27	0.036330
8	0.153178	28	0.044363
9	0.080535	29	0.025101
10	-0.001974	30	-0.001109
11	0.067497	31	0.023522
12	0.104994	32	0.038904
13	0.077124	33	0.029815
14	-0.014197	34	-0.006749
15	-0.023018	35	-0.010757
16	0.048990	36	0.021334
17	0.048636	37	0.022036
18	-0.055238	38	-0.025957
19	-0.001166	39	-0.000688
20	-0.139591	40	-0.074395
21	-0.001749	41	-0.000695

$T(9,5), T(3,2)$			
N	Difference	N	Difference
2	-0.549306	22	0.027801
3	0.578460	23	0.072810
4	0.322189	24	0.078980
5	0.353723	25	0.091445
6	0.119724	26	0.027635
7	0.296152	27	-0.009615
8	0.174782	28	0.055107
9	-0.096595	29	0.057092
10	-0.053712	30	-0.010508
11	0.153095	31	0.010443
12	0.196868	32	0.065054
13	0.148664	33	0.063422
14	0.081783	34	-0.000516
15	0.148781	35	0.032248
16	0.132714	36	-0.072707
17	0.088021	37	0.027468
18	0.003240	38	-0.006864
19	0.066780	39	0.044435
20	0.132706	40	0.055595
21	0.086341	41	0.022374

$T(3,5), T(3,2)$			
N	Difference	N	Difference
42	-0.023439	70	-0.000367
43	0.018945	71	0.010336
44	0.017438	72	0.017326
45	-0.008351	73	0.013498
46	-0.004966	74	-0.003045
47	0.020938	75	-0.004973
48	0.025950	76	0.010074
49	0.014911	77	0.010542
50	-0.000624	78	-0.012744
51	0.014327	79	-0.000410
52	0.023952	80	-0.036286
53	0.018569	81	-0.000420
54	-0.004162	82	-0.012029
55	-0.006755	83	0.009812
56	0.013699	84	0.009148
57	0.014280	85	-0.004352
58	-0.016973	86	-0.002586
59	-0.000572	87	0.011352
60	-0.048257	88	0.014197
61	-0.000483	89	0.008255
62	-0.016006	90	-0.000298
63	0.012936	91	0.008067
64	0.012016	92	0.013568
65	-0.005687	93	0.010602
66	-0.003369	94	-0.002404
67	0.014742	95	-0.003937
68	0.018370	:	:
69	0.010649	:	:

$T(9,5), T(3,2)$			
N	Difference	N	Difference
42	0.016612	70	-0.019148
43	0.050636	71	0.005889
44	0.029700	72	-0.043068
45	-0.002854	73	0.011614
46	0.017939	74	-0.002688
47	0.047056	75	0.028207
48	0.049100	76	0.020148
49	0.029444	77	0.020037
50	-0.017061	78	-0.000626
51	0.027716	79	0.019452
52	0.046837	80	0.019952
53	0.022438	81	-0.008948
54	-0.008414	82	0.001369
55	0.024691	83	0.025068
56	0.034838	84	0.022553
57	0.033579	85	0.022575
58	0.012023	86	0.012937
59	0.028589	87	0.025128
60	0.041932	88	0.028882
61	0.030457	89	0.016187
62	0.008302	90	0.002406
63	-0.005312	91	0.015826
64	0.018421	92	0.027617
65	0.035618	93	0.023492
66	0.014242	94	0.011815
67	0.025116	95	0.020180
68	0.029356	:	:
69	0.019037	:	:

Graph3

6 Results of computing

These results of computing are following.

- At the present time, our conjecture does not hold.
- The period can be observed.

But, we need to compute the colored Jones polynomial for large N and other partial order.
Now, we compute colored Jones polynomial for a 2-bridge knot. If we can compute, our conjecture may hold and existence of period may be proved.

Thank you!