On amphicheiral links

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Contents

§1. Symmetries of links

Amphicheiral link

L **: (oriented) link**

L^* **: (oriented) mirror image of** L

- L **:** <u>**amphicheiral**</u> $\Longleftrightarrow L \cong L^*$ **as unoriented links**

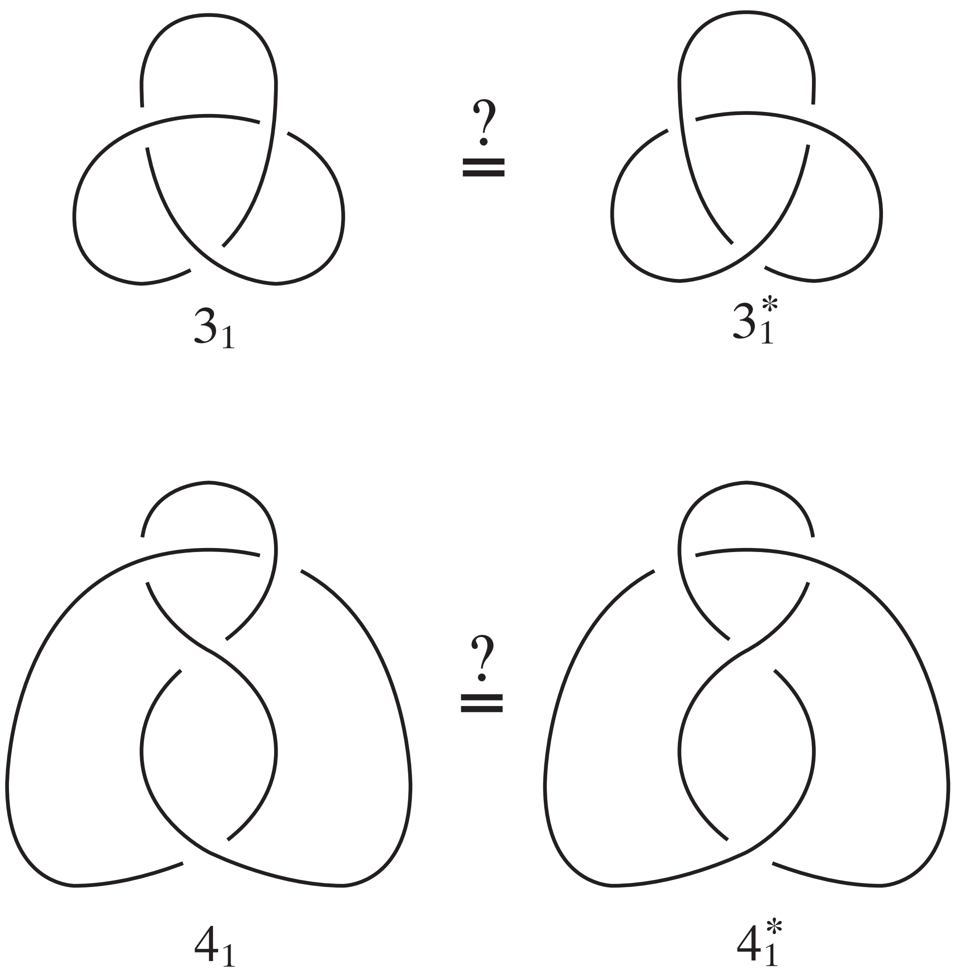


Figure 1: unoriented mirror image

◦ K **: oriented knot**

$K \sharp K^*, \; K \sharp (-K^*)$ **: amphicheiral knots**

Invertible link

L **: oriented link**

$-L$ **: orientation-reversed link of** L

- L **:** **invertible** $\Longleftrightarrow L \cong -L$ **as oriented links**

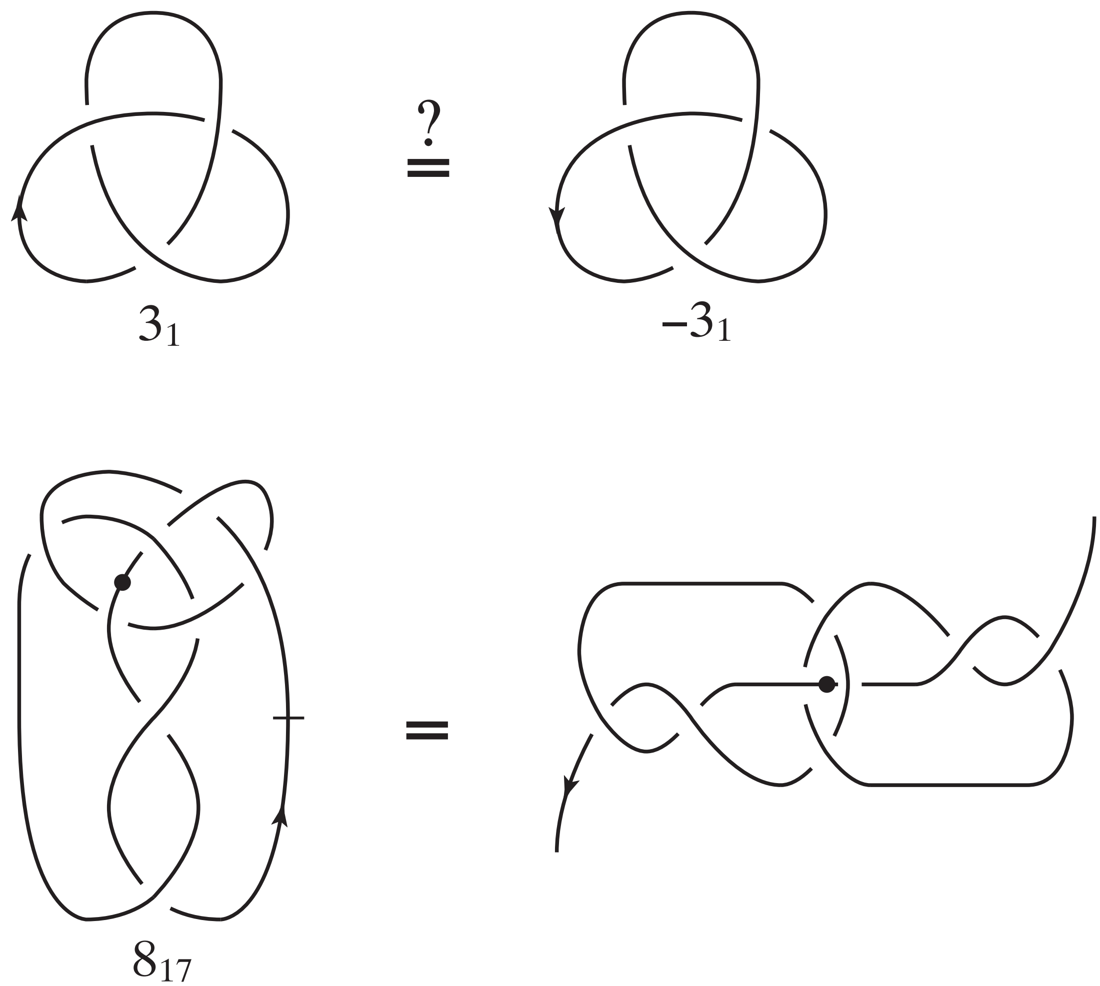


Figure 2: orientation-reversed link

Question **Is** 8_{17} **invertible ?**

∘ K **: oriented knot**

$K\sharp(-K)$ **: invertible knot**

Link symmetric group

- G_r **:** **r-th universal $(+)$-link symmetric group**

  $1 \to (\mathbb{Z}/2\mathbb{Z})^r \to G_r \to \mathfrak{S}_r \to 1$ **: split exact**

  $G_r \cong (\mathbb{Z}/2\mathbb{Z})^r \,\widetilde{\times}\, \mathfrak{S}_r$ **: semi-product**

  $\mathfrak{S}_r$ **:** **r-th symmetric group**

  $\varepsilon_i, \eta_j \in \mathbb{Z}/2\mathbb{Z} = \{1, -1\} = \{+, -\};\ \sigma, \tau \in \mathfrak{S}_r$

  $\mathbf{x} = (\varepsilon_1, \ldots, \varepsilon_r; \sigma),\ \mathbf{y} = (\eta_1, \ldots, \eta_r; \tau) \in G_r,$

  $\mathbf{x} \cdot \mathbf{y} := (\varepsilon_1 \eta_{\sigma^{-1}(1)}, \ldots, \varepsilon_r \eta_{\sigma^{-1}(r)}; \sigma \circ \tau)$

- ◦ $1 = +1 = +,\ -1 = -$ **in** $\mathbb{Z}/2\mathbb{Z}$,

  ι **: identity in** $\mathfrak{S}_r$

- ◦ $(+, \ldots, +; \iota)$ **: unit in** G_r.

- Γ_r **:** **r-th universal link symmetric group**

  $\Gamma_r \cong \mathbb{Z}/2\mathbb{Z} \times G_r$

- ◦ $\kappa,\ \varepsilon_i \in \mathbb{Z}/2\mathbb{Z};\ \sigma \in \mathfrak{S}_r$

  $\mathbf{x} = \kappa(\varepsilon_1, \ldots, \varepsilon_r; \sigma) \in \Gamma_r$

- ◦ Γ_r **includes information of amphicheirality, invertibility and interchangeability for all r-component links.**

- ◦ **By using natural inclusions**

  $(\mathbb{Z}/2\mathbb{Z})^r \subset (\mathbb{Z}/2\mathbb{Z})^{r+1}$ **and** $\mathfrak{S}_r \subset \mathfrak{S}_{r+1}$,

  we can define $G_\infty = \varinjlim_r G_r$ **and** $\Gamma_\infty = \varinjlim_r \Gamma_r$.

- $L = K_1 \cup \ldots \cup K_r$

  : r**-component oriented ordered link in** S^3

  L **:** $\underline{\kappa(\varepsilon_1, \ldots, \varepsilon_r; \sigma)\textbf{-symmetric}}$ $\Longleftrightarrow$

  ${}^{\exists}\varphi : S^3 \to S^3$ **:** (κ)**-homeomorphism**

  $$\kappa = \begin{cases} + & (\varphi \textbf{ : orientation-preserving}), \\ - & (\varphi \textbf{ : orientation-reversing}). \end{cases}$$

  s.t. $\varphi(K_i) = \varepsilon_{\sigma(i)} K_{\sigma(i)}$ **for** $i = 1, \ldots, r$

  where $\varepsilon_i \in \{+, -\}$ **and** $\sigma \in \mathfrak{S}_r$.

- ◦ $\kappa(\varepsilon_1, \ldots, \varepsilon_r; \sigma) \in \Gamma_r$
- ◦ $\kappa(\varepsilon_1, \ldots, \varepsilon_r; \sigma)$**-symmetric**

  $$= \begin{cases} (\varepsilon_1, \ldots, \varepsilon_r; \sigma)\textbf{-invertible} & (\kappa = +), \\ (\varepsilon_1, \ldots, \varepsilon_r; \sigma)\textbf{-amphicheiral} & (\kappa = -). \end{cases}$$

- ◦ **If** $\sigma = \iota$**, then we may abbreviate** ι**.**

- $L = K_1 \cup \ldots \cup K_r$

  : r**-component oriented ordered link in** S^3

  $\Gamma(L) = \{\mathbf{x} \in \Gamma_r \mid L : \mathbf{x}\text{-symmetric}\}$

  : **link symmetric group of** L

  $G(L) = \Gamma(L) \cap G_r$

  : $(+)$**-link symmetric group of** L

◦ $\Gamma(L)$ **is a subgroup of** Γ_r **which is stable up to conjugations induced by changing orders and orientations.**

◦ $\Gamma(L)$ **is not always a normal subgroup of** Γ_r**.**

Problem 1 **For a link** L**, determine** $\Gamma(L)$.

Problem 2 **For a subgroup** $H \subset \Gamma_r$**, does there exist a link** L **realizing** $H = \Gamma(L)$ **?**

◦ L **:** **invertible** $\Longleftrightarrow$

  L **:** $^{\exists}(\varepsilon_1, \ldots, \varepsilon_r; \sigma)$**-invertible &** $^{\exists}i,\ \varepsilon_i = -$.

  The original definition of "invertible" was only $(-, \ldots, -; \sigma)$**-invertible.**

◦ L **:** **amphicheiral** $\Longleftrightarrow \Gamma(L) \setminus G(L) \neq \emptyset$.

- $\circ$ L **:** **<u>component-preservingly invertible</u>** $\Longleftrightarrow$

  L **:** $^{\exists}(\varepsilon_1, \ldots, \varepsilon_r; \iota)$**-invertible &** $^{\exists}i,\ \varepsilon_i = -$.
- $\circ$ L **:** **<u>component-preservingly amphicheiral</u>** $\Longleftrightarrow$

  L **:** $^{\exists}(\varepsilon_1, \ldots, \varepsilon_r; \iota)$**-amphicheiral.**
- $\circ$ L **:** <u>$(-)$**-invertible**</u> $\Longleftrightarrow$ L **:** $(-, \ldots, -; \sigma)$**-invertible**

  by some orientation of L**.**
- $\circ$ L **:** <u>(ε)**-amphicheiral**</u> $\Longleftrightarrow$ L **:** $(\varepsilon, \ldots, \varepsilon; \sigma)$**-amphicheiral**

  by some orientation of L**.**

ex.1 (1) $G_1 \cong \mathbb{Z}/2\mathbb{Z},\ \Gamma_1 \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$.

K **: oriented knot**

K **: invertible** $\Longleftrightarrow G(K) = G_1 \Longleftrightarrow (+, (-)) \in \Gamma(K)$.

K **:** $(+)$**-amphicheiral** $\Longleftrightarrow (-, (+)) \in \Gamma(K)$.

K **:** $(-)$**-amphicheiral** $\Longleftrightarrow (-, (-)) \in \Gamma(K)$.

Two properties generate the rest.

Since 8_{17} **is** $(-)$**-amphicheiral,**

“8_{17} **is invertible** $\Longleftrightarrow$ 8_{17} **is** $(+)$**-amphicheiral”**

(2) K **: oriented knot**

$K \sharp K$	:	2**-periodic**
$K \sharp (-K)$	:	**invertible**
$K \sharp K^*$	:	$(+)$**-amphicheiral**
$K \sharp (-K^*)$	:	$(-)$**-amphicheiral & slice**

(3) $G_2 \cong (\mathbb{Z}/2\mathbb{Z})^2 \mathbin{\widetilde{\times}} \mathbb{Z}/2\mathbb{Z} \cong D_4,\ \ \Gamma_2 \cong \mathbb{Z}/2\mathbb{Z} \times D_4.$

$a = (-,+;(1\ 2)),\ \ b = (+,+;(1\ 2)) \in G_2$

$G_2 = \langle a, b \mid a^4 = b^2 = 1,\ \ b^{-1}ab = a^{-1} \rangle \cong D_4.$

- $\Psi_r : \Gamma_r \to \mathbb{Z}/2\mathbb{Z} = \{1, -1\};$ **surjective**

$$\Psi_r(\kappa(\varepsilon_1, \ldots, \varepsilon_r; \sigma)) = \rho \cdot \prod_{i=1}^{r} \varepsilon_i \cdot \mathrm{sgn}\,(\sigma)$$

where $\rho = \begin{cases} 1 & (\kappa = 1), \\ (-1)^{r-1} & (\kappa = -1). \end{cases}$

$\Omega_r = \mathrm{Ker}\,(\Psi_r) \subset \Gamma_r$ **: (no name; Ω-subgroup)**

ex.2 (1) $\Omega_1 = \{(+,(+)), (-,(+))\} \cong \mathbb{Z}/2\mathbb{Z}$

(2) $a = (+,(-,+;(1\ 2))),\ \ b = (-,(+,+;(1\ 2))) \in \Gamma_2$

$\Omega_2 = \langle a, b \mid a^4 = b^2 = 1,\ \ b^{-1}ab = a^{-1} \rangle \cong D_4.$

(3) $a = (+,+,+;(1\ 2\ 3)),\ \ b = (-,+,+;(1\ 2))) \in G_3$

$\Omega_3 \cap G_3 = \langle a, b \mid a^3 = b^4 = (ab)^2 = 1 \rangle \cong \mathfrak{S}_4.$

$\Omega_3 \cong \mathbb{Z}/2\mathbb{Z} \times \mathfrak{S}_4.$

• $\overline{\mu}_L(\cdots)$ **: Milnor's $\overline{\mu}$-invariant**

◦ $\overline{\mu}_L(ij) = \mathrm{lk}\,(K_i, K_j)\ (i \neq j)$

Lemma 1 $r \geq 2,\ \overline{\mu}_L(12\cdots r) \neq 0 \Longrightarrow \Gamma(L) \subset \Omega_r \subset \Gamma_r.$

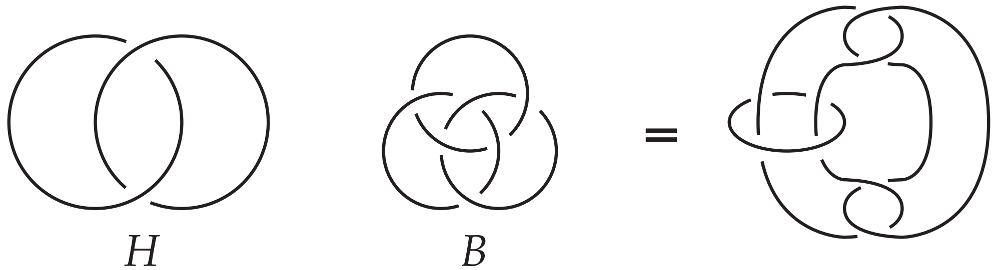


Figure 3: Hopf link and Borromean rings

ex.3 (1) $H = K_1 \cup K_2$ **: Hopf link**

$\Gamma(H) = \Omega_2.$

(2) $B = K_1 \cup K_2 \cup K_3$ **: Borromean rings**

$\Gamma(B) = \Omega_3.$

Milnor link

- $M_\lambda = K_1 \cup \cdots \cup K_\lambda$ **:** λ**-component Milnor link**

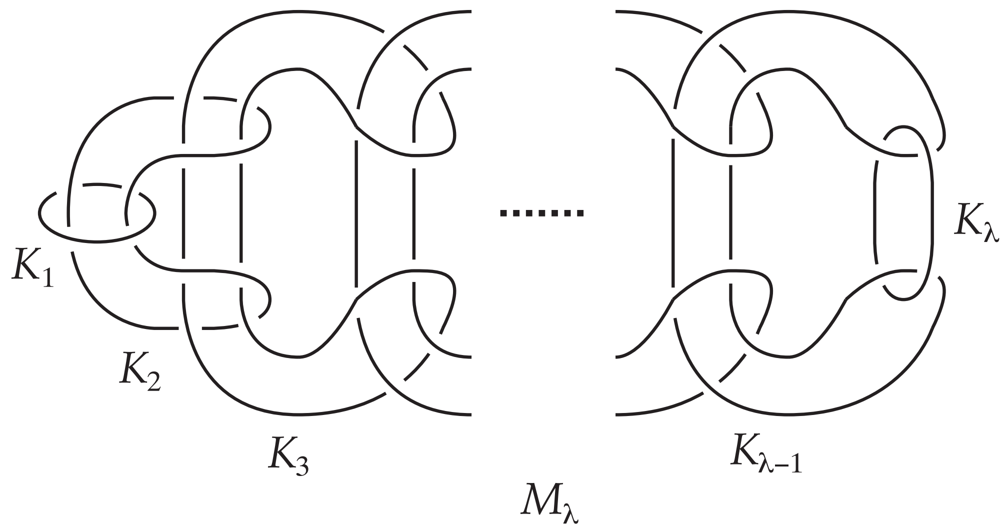


Figure 4: λ-component Milnor link M_λ

- ∘ $M_2 = H$ **and** $M_3 = B$.

- ∘ $^\forall$ **proper sublink of** M_λ **: trivial link**
  i.e. M_λ **: Brunnian link**
  In particular, algebraically split link.

- ∘ M_λ **: component-preservingly amphicheiral**
  & component-preservingly invertible

- ∘ $\{K_1,\ K_2\},\ \{K_{\lambda-1},\ K_\lambda\}$ **: interchangeable**

- ∘ **The order** $(K_1, K_2, \ldots, K_\lambda)$ **can be changed**
  into $(K_\lambda, \ldots, K_2, K_1)$ **by (ori.-pres.) homeo.**

Chain link

- C **: positive clasp tangle**

  C^* **: negative clasp tangle**

  T_k **:** 2**-string tangle with** k **half twists**

  $L = C^n T_k$ **:** <u>n**-chain link**</u>

◦ $C^* = CT_1 = T_1C$ **and** $C = C^*T_{-1} = T_{-1}C^*$.

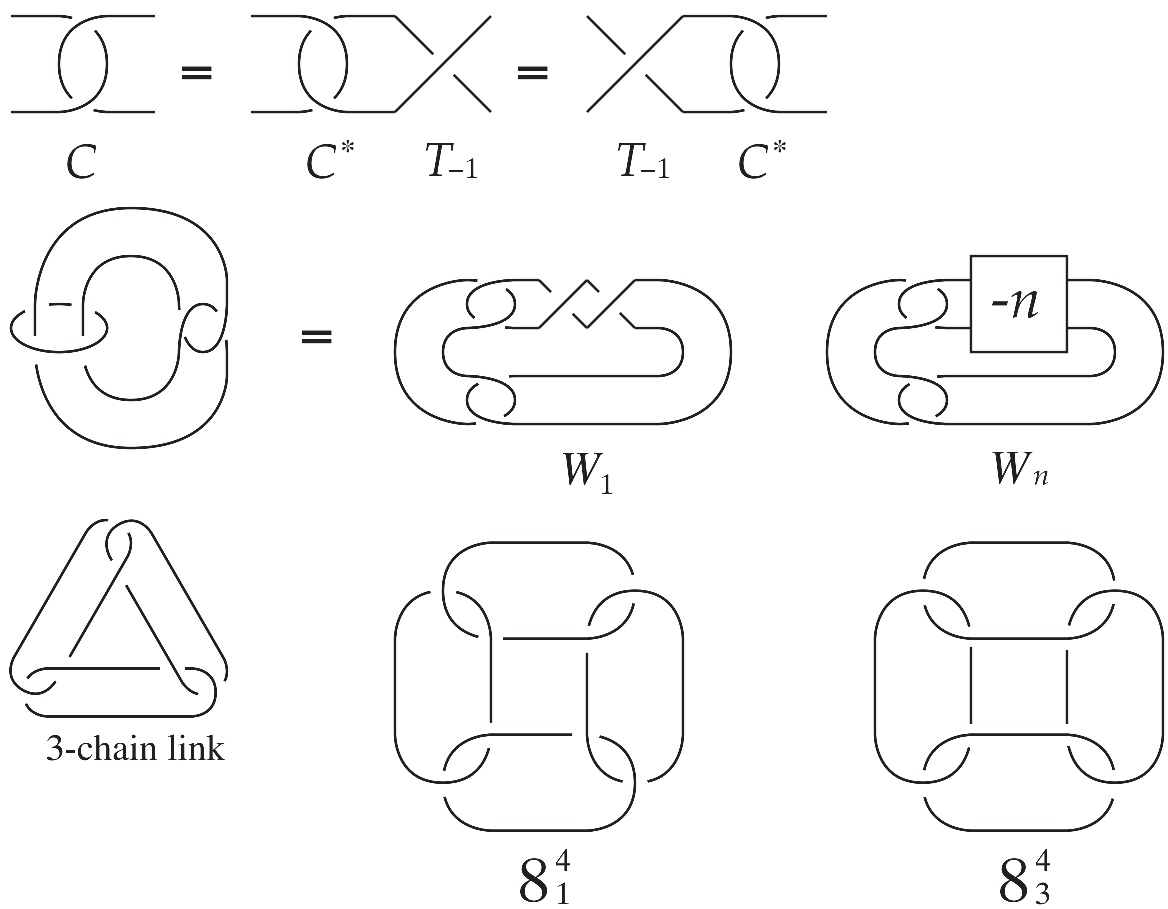


Figure 5: chain links

- $p = \dfrac{n}{2} - k$.

  $L = C^n T_k$ **:** <u>p**-twisted** n**-chain link**</u>**.**

- $p = 0 \Longleftrightarrow L$ **:** <u>**untwisted chain link**</u>**.**

  $p \neq 0 \Longleftrightarrow L$ **:** <u>**twisted chain link**</u>**.**

- n **: odd** $\Longrightarrow L$ **: twisted chain link.**

- L **: untwisted chain link** $\Longleftrightarrow L = (CC^*)^{\frac{n}{2}}$.

- **An untwisted chain link is amphicheiral.**

Question **Can twisted chain links be amphicheiral ?**

- $W_n = CC^*T_{-n} = C^2T_{-n+1}$ **:** n**-twisted** 2**-chain link.**

- $8_1^4 = C^4$ **:** 2**-twisted** 4**-chain link.**

- $8_3^4 = (CC^*)^2$ **: untwisted** 4**-chain link.**

§2. Conditions from Alexander polynomials

Alexander polynomial

$L = K_1 \cup \ldots \cup K_r$ **: (oriented)** r**-component link in** S^3

$\Delta_L(t_1, \ldots, t_r) \in \mathbb{Z}[t_1^{\pm 1}, \ldots, t_r^{\pm 1}]$

: r**-variable** **<u>Alexander polynomial</u> of** L

- ∘ $\Delta_L(t_1, \ldots, t_r)$ **is determined up to multiplication of** $\pm t_1^{m_1} \cdots t_r^{m_r}$ $(m_i \in \mathbb{Z})$.
- • $A \doteq B \Longleftrightarrow A = \pm t_1^{m_1} \cdots t_r^{m_r} B$ $(m_i \in \mathbb{Z})$.
- ∘ **(duality)** $\Delta_L(t_1, \ldots, t_r) \doteq \Delta_L\left(t_1^{-1}, \ldots, t_r^{-1}\right)$.

(Torres condition)

(i) $r = 1$; $|\Delta_L(1)| = 1$.

(ii) $r = 2$; $\Delta_L(t_1, 1) \doteq \dfrac{t_1^\ell - 1}{t_1 - 1} \cdot \Delta_{K_1}(t_1)$ **where** $\ell = \mathrm{lk}(L)$.

$|\Delta_L(1,1)| = \ell$.

$\ell = 0 \Longrightarrow \Delta_L(t_1, 1) = \Delta_L(1, t_2) = 0$

$\Longrightarrow \Delta_L(t_1, t_2) \doteq (t_1 - 1)(t_2 - 1) f(t_1, t_2)$.

(iii) $r \geq 3$; $L' = K_1 \cup \ldots \cup K_{r-1}$, $\ell_i = \mathrm{lk}(K_i, K_r)$

$$\Delta_L(t_1, \ldots, t_{r-1}, 1) \doteq \left(t_1^{\ell_1} \cdots t_{r-1}^{\ell_{r-1}} - 1\right) \cdot \Delta_{L'}(t_1, \ldots, t_{r-1}).$$

- ∘ L **: split link** $\Longrightarrow \Delta_L(t_1, \ldots, t_r) = 0$.
- ∘ L **: algebraically split**

$\Longrightarrow \Delta_L(t_1, \ldots, t_r) \doteq (t_1 - 1) \cdots (t_r - 1) f(t_1, \ldots, t_r)$.

◦ **It looks that the Alexander polynomial of a knot does <u>not</u> distinguish amphicheirality.**

ex.4 O_r **:** r**-component trivial link,** $O_1 = O$.

$\Delta_O(t) \doteq 1$

$\Delta_{O_r}(t_1, \ldots, t_r) = 0 \ (r \geq 2)$

$\Delta_{3_1}(t) \doteq t^2 - t + 1$

$\Delta_{4_1}(t) \doteq t^2 - 3t + 1$

$\Delta_H(t_1, t_2) \doteq 1$

$\Delta_B(t_1, t_2, t_3) \doteq (t_1 - 1)(t_2 - 1)(t_3 - 1)$

W_n **:** n**-twisted Whitehead link,** $W_1 = W$.

$\Delta_{W_n}(t_1, t_2) \doteq n(t_1 - 1)(t_2 - 1)$

$\Delta_W(t_1, t_2) \doteq (t_1 - 1)(t_2 - 1)$

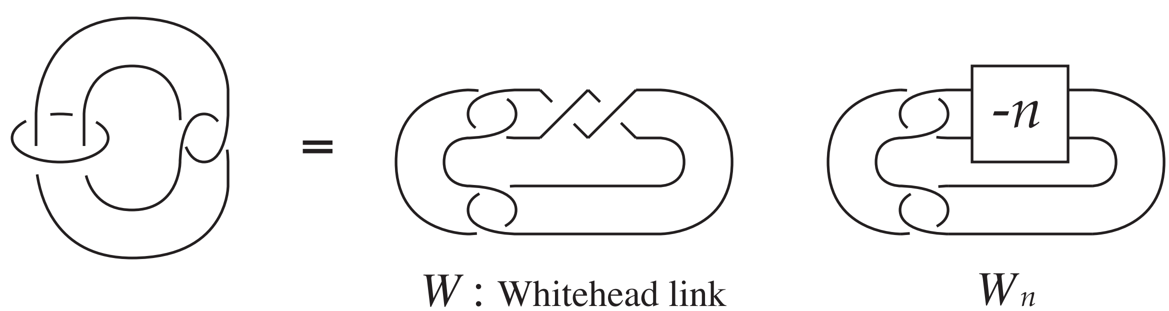


Figure 6: n-twisted Whitehead link W_n

$\Delta_{8_{17}}(t) \doteq t^6 - 4t^5 + 8t^4 - 11t^3 + 8t^2 - 4t + 1$

Lemma 1 $L = K_1 \cup \ldots \cup K_r$

: $(\varepsilon_1, \ldots, \varepsilon_r; \sigma)$**-amphicheiral/invertible link**

$$\Delta_L(t_1, \ldots, t_r) \doteq \Delta_L\left(t_{\sigma(1)}^{\varepsilon_{\sigma(1)}}, \ldots, t_{\sigma(r)}^{\varepsilon_{\sigma(r)}}\right).$$

- **Useless for component-preservingly** (ε)**-amphicheiral/invertible links**
- **A component-preservingly** (ε)**-amphicheiral link is algebraically split.**

Theorem 2 **[Hartley-Kawauchi]**

(1) K **:** $(-)$**-amphicheiral knot**

$\Longrightarrow \exists f(t) \in \mathbb{Z}[t]$ **s.t.** $f(t^{-1}) \doteq f(-t)$ & $|f(1)| = 1$ &

$\Delta_K\left(t^2\right) \doteq f(t)f\left(t^{-1}\right) \doteq f(t)f(-t)$.

(2) K **:** $(+)$**-amphicheiral knot**

$\Longrightarrow \exists r_j(t) \in \mathbb{Z}[t]$ **: type** X**,**

$\exists \alpha_j$ **: odd** $(j = 1, \ldots, m)$ **s.t.**

$$\Delta_K(t) \doteq \prod_{j=1}^{m} r_j\left(t^{\alpha_j}\right).$$

In particular, if K **: hyperbolic, we can take** $m = \alpha_1 = 1$.

- $r(t) \in \mathbb{Z}[t]$ **:** **<u>type X</u>** $\Longleftrightarrow$

  $\exists n \geq 0$ **&** $\lambda \geq 3$ **: odd &** $g_i(t) \in \mathbb{Z}[t]$ $(i = 0, 1, \ldots, n)$

  s.t. $g_i(t) \doteq g_i\left(t^{-1}\right)$ **&** $|g_i(1)| = 1$ **&**

  for $i > 0,\ g_i(t) \doteq g_0(t)^{2^i} p_\lambda(t)^{2^{i-1}} \pmod 2$ **where**

  $$p_\lambda(t) = \frac{t^\lambda - 1}{t - 1} \text{ \& } r(t) \doteq \begin{cases} g_0(t)^2 & (n = 0), \\ g_0(t)^2 g_1(t) \cdots g_n(t) & (n \geq 1). \end{cases}$$

Corollary 3 K **:** $(-)$**-amphicheiral knot**

$|\Delta_K(-1)| = p_1^{r_1} \cdots p_m^{r_m}$ **: prime factorization**

$p_i \equiv 3 \pmod 4 \Longrightarrow r_i$ **: even.**

ex.5 (1) $|\Delta_{3_1}(-1)| = 3 \Longrightarrow 3_1$ **: non-amphicheiral.**

(2) $\Delta_{4_1}\left(t^2\right) \doteq (t^2 + t - 1)(t^2 - t - 1)$

$f(t) = t^2 + t - 1$

$m = \alpha_1 = 1,\ n = 1,\ \lambda = 3,\ g_0(t) = 1.$

4_1 **:** $(\pm)$**-amphicheiral.**

(3) $\Delta_{8_{17}}\left(t^2\right) \doteq (t^6 - 2t^4 + t^3 + 2t^2 - 1)(t^6 - 2t^4 - t^3 + 2t^2 - 1).$

$f(t) = t^6 - 2t^4 + t^3 + 2t^2 - 1$

8_{17} **:** $(-)$**-amphicheiral & hyperbolic**

$\Longrightarrow m = \alpha_1 = 1$ **&** $\Delta_{8_{17}}(t) \equiv (t^2 + t + 1)^3 \pmod 2$

$\Longrightarrow n = 2,\ \lambda = 3,\ g_0(t) = 1$ **&** $(t^2 - t + 1) \not| \Delta_{8_{17}}(t)$

$\Longrightarrow$ **non-**$(+)$**-amphicheiral & non-invertible.**

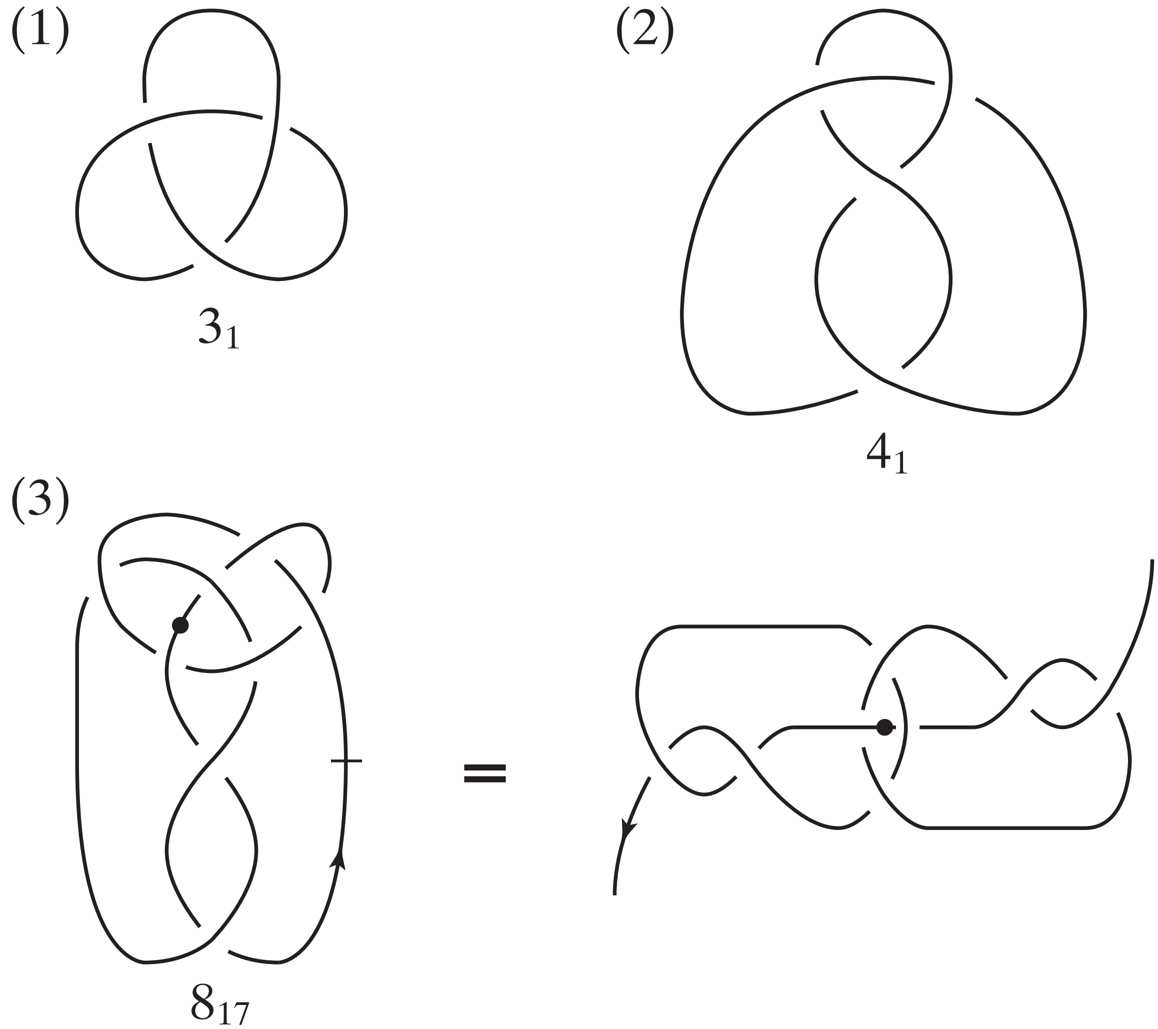


Figure 7: Examples

Conditions from linking numbers

Lemma 4 $L = K_1 \cup \ldots \cup K_r,\ \ell_i = \mathrm{lk}(K_i, K_{i+1})\ (K_{r+1} = K_1)$

(1) r **: odd &** $\forall \ell_i \neq 0$

$\Longrightarrow L$ **: not component-preservingly amphicheiral.**

(2) $r = 3$ **&** $\forall \ell_i \neq 0$

$\Longrightarrow L$ **: non-amphicheiral.**

Theorem 5 $L = K_1 \cup K_2$ **with** $\ell = \mathrm{lk}(L)$.

(1) **[Hartley]**

L **: component-preservingly amphicheiral**

$\Longrightarrow \ell = 0$ **or odd.**

(2) L **:** $(\varepsilon, -\varepsilon; (1\ 2))$**-amphicheiral** $\Longrightarrow \ell \not\equiv 2 \pmod 4$.

◦ **linking graph**

Main Theorems

Conjecture
L **:** r**-component algebraically split component-preservingly amphicheiral link with** r **: even** $\Longrightarrow \Delta_L(t_1, \ldots, t_r) = 0$.

Lemma 6 L **:** r**-component algebraically split component-preservingly** (ε)**-amphicheiral link with** r **: even** $\Longrightarrow \Delta_L(t^{\eta_1}, \ldots, t^{\eta_r}) = 0$ **with** $\eta_i \in \{\pm\}$.

Lemma 7 L **:** 2**-component algebraically split component-preservingly amphicheiral link**
$\Longrightarrow (t_1 - 1)^2(t_2 - 1)^2 | \Delta_L(t_1, t_2)$.

Lemma 8 L **:** 2**-component algebraically split** (ε)**-amphicheiral link**
$\Longrightarrow (t_1 - 1)^2(t_2 - 1)^2(t_1 t_2 - 1)(t_1 - t_2) | \Delta_L(t_1, t_2)$.

Main Theorem 1 [K-Kawauchi]
$L = K_1 \cup \ldots \cup K_r$
: r**-component amphicheiral link**
$\ell_{ij} = \mathrm{lk}(K_i, K_j)$
$$r + \sum_{1\le i<j\le r} \ell_{ij} \textbf{ : even} \Longrightarrow \Delta_L(-1,\ldots,-1) = 0.$$

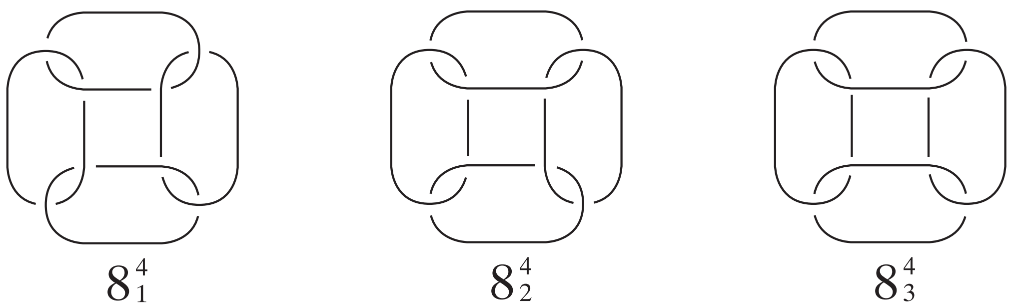


Figure 8: Chain links

- $\Delta_{8_1^4}(-1,-1,-1,-1) = \pm 16$
  $\Delta_{8_2^4}(-1,-1,-1,-1) = \pm 16$
  $\Delta_{8_3^4}(-1,-1,-1,-1) = 0.$

Main Theorem 2 [K-Kawauchi]
L **:** r**-component component-preservingly**
(ε)**-amphicheiral with** r **: even**
$\Longrightarrow \Delta_L(t_1,\ldots,t_r) = 0.$

- **A partial affirmative answer for the conjecture.**
  We want to remove "(ε)**-".**

§3. Conditions from Jones polynomials

$L = K_1 \cup \ldots \cup K_r$ **: oriented r-component link in** S^3

$V_L(t) \in \mathbb{Z}[t^{\pm 1/2}]$ **: Jones polynomial of** L

Lemma 9 L **: amphicheiral**
$\Longrightarrow$ **Coefficients of** $V_L(t)$ **are symmetric.**

ex.6 (1) $V_{3_1}(t) = -t^4 + t^3 + t \Longrightarrow 3_1$ **: non-amphicheiral.**

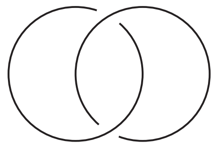
(2) $V_{4_1}(t) = t^2 - t + 1 - t^{-1} + t^{-2}$.

(3) **Coefficients of** $V_{W_n}(t)$ $(n \neq 0)$ **are not symmetric.**

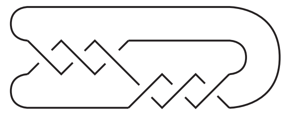
(4) $V_B(t) = -t^3 + 3t^2 - 2t + 4 - 2t^{-1} + 3t^{-2} - t^{-3}$.

§4. Prime amphicheiral links with up to 11 crossings

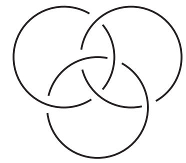
Table 1/3 [K-Kawauchi]



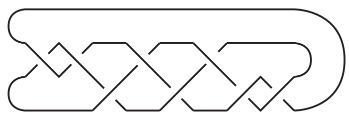
2_1^2



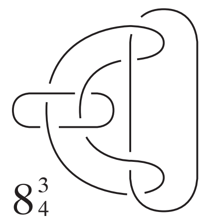
6_2^2



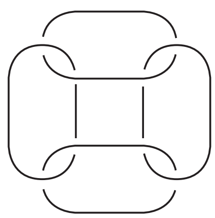
6_2^3



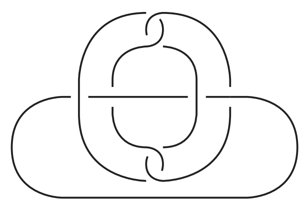
8_8^2



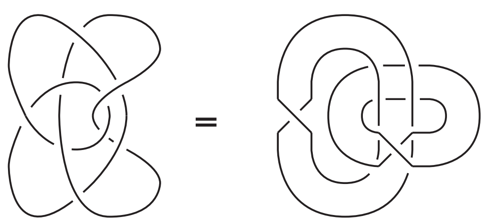
8_4^3



8_3^4



8_6^3



9_{61}^2

Figure 9: Prime amphicheiral link with up to 11 crossings I

Table 2/3 [**K**]

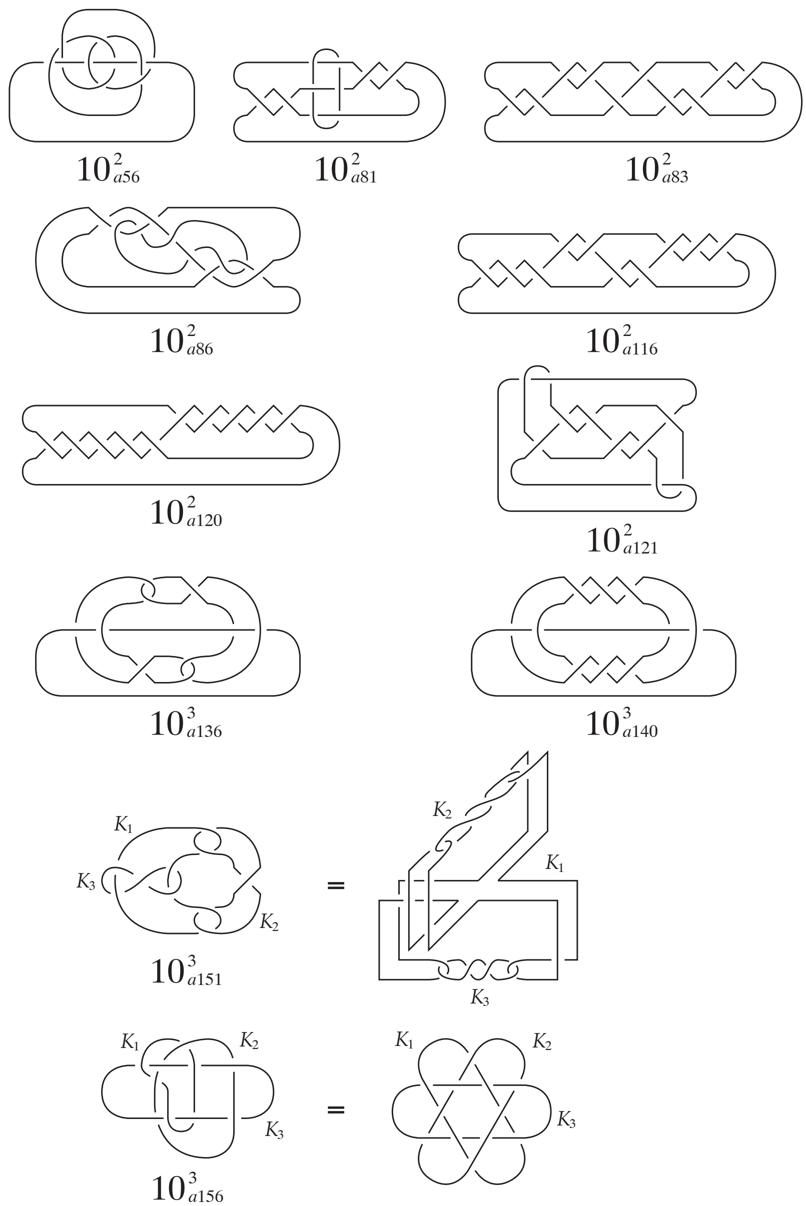


Figure 10: Prime amphicheiral link with up to 11 crossings II

Table 3/3 [**K**]

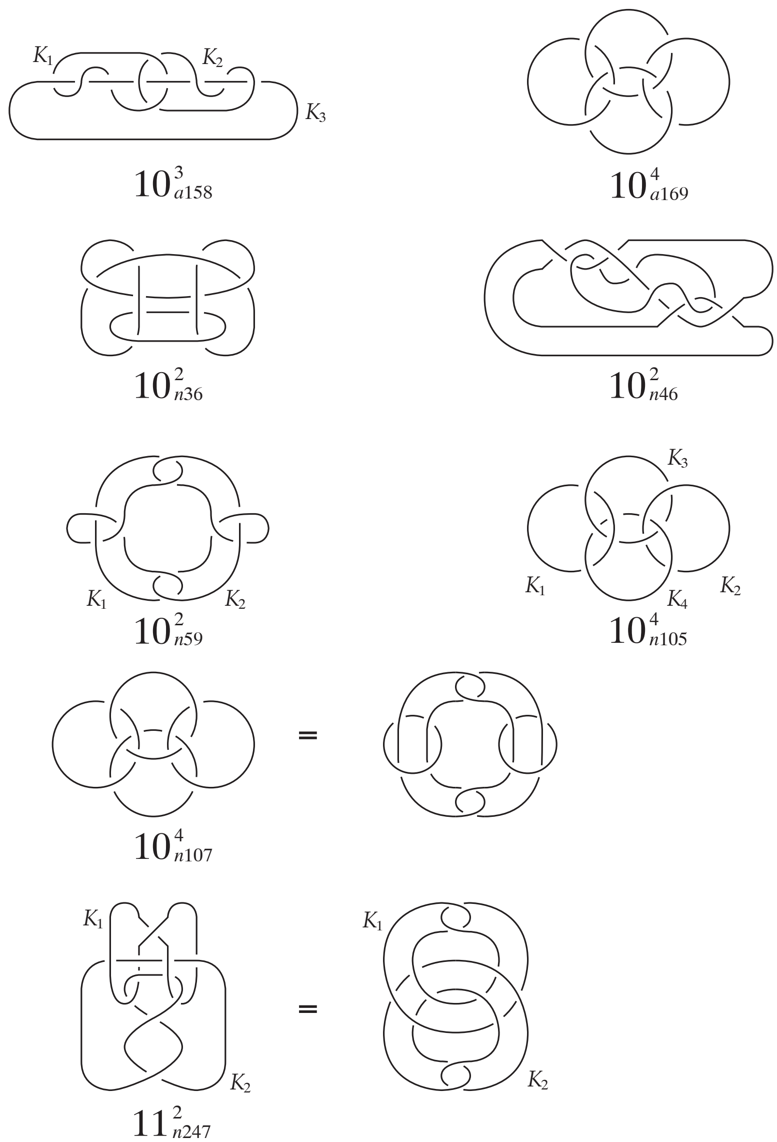


Figure 11: Prime amphicheiral link with up to 11 crossings III

§5. Component-preservingly amphicheiral links with odd crossings

[Kobatake]

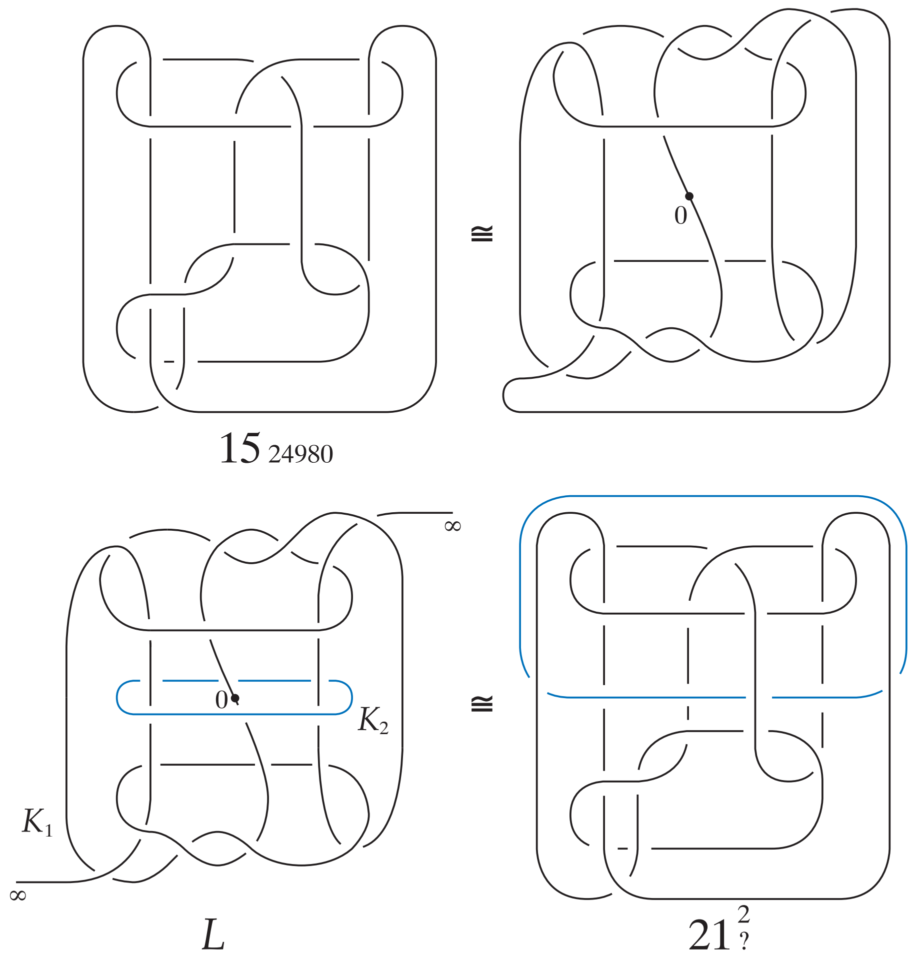


Figure 12: Component-preservingly amphicheiral link with odd crossings

§6. Construction of amphicheiral links

- **generalized Murasugi sum**
  tangle decomposition [Kobatake]

Conjecture
Every amphicheiral link has Kobatake's tangle decomposition.

◦ **"True" for links with up to 11 crossings and for knots with up to 12 crossings. [Kobatake]**